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Senior School Certificate Examination

July — 2015 (Comptt.)

Marking Scheme — Mathematics (Delhi) 65/1/1, 65/1/2, 65/1/3

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question (s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65/1/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

		Marks
1.	$\hat{a} = \frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}$ then $7\hat{a} = \frac{7}{\sqrt{5}}\hat{i} - \frac{14}{\sqrt{5}}\hat{j}$	½ + ½
2.	$(\sqrt{2}\vec{a} - \vec{b}) \cdot (\sqrt{2}\vec{a} - \vec{b}) = 1 \Rightarrow \theta = \frac{\pi}{4}$	½ + ½
3.	$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$	½ + ½
4.	$AB = \begin{bmatrix} -1 & 5 \\ 4 & 8 \end{bmatrix} \Rightarrow \lvert AB \rvert = -28$	½ + ½
5.	$1 \cdot y + x\frac{dy}{dx} = -c\sin x \Rightarrow x\frac{dy}{dx} + y + xy\tan x = 0$	½ + ½
6.	order = 2, degree = 3, sum = 2 + 3 = 5	½ + ½

SECTION B

7. System of equation is

3x + y + 2z = 1100, x + 2y + 3z = 1400, x + y + z = 600 — 1½

(i) Matrix equation is

$$\begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1100 \\ 1400 \\ 600 \end{bmatrix}$$ — 1

(ii) |A| = –3 ≠ 0, system of equations can be solved. — ½

(iii) Any one value with reason. — 1

8. $\begin{bmatrix} 2x & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = 0$

$\begin{bmatrix} 2x-9 & 4x \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = 0$ 1

$[2x^2 - 9x + 12x] = [0] \Rightarrow 2x^2 + 3x = 0,\ x = 0 \text{ or } \frac{-3}{2}$ 1+1+1

9. $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$

$= \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ 4a+10 & 2 & 0 \end{vmatrix} \left.\begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}\right\}$ 1+1

$= 4a + 8 - 4a - 10 = -2.$ 1+1

10. $I = \int_0^{\pi/2} \frac{1}{1+\sqrt{\tan x}}\, dx$

$I = \int_0^{\pi/2} \frac{1}{1+\sqrt{\tan(\pi/2 - x)}}\, dx = \int_0^{\pi/2} \frac{1}{1+\sqrt{\cot x}}\, dx = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\, dx$ 1½

$\Rightarrow 2I = \int_0^{\pi/2} \frac{1+\sqrt{\tan x}}{1+\sqrt{\tan x}}\, dx = \int_0^{\pi/2} 1 \cdot dx = \frac{\pi}{2}$ 1½

$\Rightarrow I = \frac{\pi}{4}$ 1

11. $\frac{x}{(x-1)^2\,(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$ ½

$A = \frac{2}{9},\ B = \frac{1}{3},\ C = -\frac{2}{9}$ 1½

$$\int \frac{x}{(x-1)^2\ (x+2)}dx = \int \frac{2}{9(x-1)}dx + \int \frac{1}{3(x-1)^2}dx - \int \frac{2}{9(x+2)}dx$$ 1½

$$= \frac{2}{9}\log|x-1| - \frac{1}{3(x-1)} - \frac{2}{9}\log|x+2| + C$$ 1½

12. Let X be the number of defective bulbs. Then

$X = 0, 1, 2$ 1

$$P(X=0) = \frac{10_{C_2}}{15_{C_2}} = \frac{3}{7},\ P(X=1) = \frac{10_{C_1}\cdot 5_{C_1}}{15_{C_2}} = \frac{10}{21}$$ 1+1

$$P(X=2) = \frac{5_{C_2}}{15_{C_2}} = \frac{2}{21}$$ 1

X	0	1	2
P(X)	$\frac{3}{7}$	$\frac{10}{21}$	$\frac{2}{21}$

OR

E_1: Problem is solved by A.

E_2: Problem is solved by B.

$$P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{3}, P(\bar{E}_1) = \frac{1}{2}, P(\bar{E}_2) = \frac{2}{3}$$ 1

$$P(E_1 \cap E_2) = P(E_1)\cdot P(E_2) = \frac{1}{6}$$

P(problem is solved) = $1 - P(\bar{E}_1)\cdot P(\bar{E}_2) = 1 - \frac{1}{2}\cdot\frac{2}{3} = \frac{2}{3}$ 1½

P(one of them is solved) = $P(E_1)P(\bar{E}_2) + P(\bar{E}_1)P(E_2)$

$$= \frac{1}{2}\cdot\frac{2}{3} + \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{2}$$ 1½

13. $\overrightarrow{AB} = -4\hat{i}-6\hat{j}-2\hat{k}$

$\overrightarrow{AC} = -\hat{i}+(\lambda-5)\,\hat{j}+3\hat{k}$ 1½

$\overrightarrow{AD} = -8\hat{i}-\hat{j}+3\hat{k}$

$$\overrightarrow{AB}\cdot(\overrightarrow{AC}\times\overrightarrow{AD}) = \begin{vmatrix} -4 & -6 & -2 \\ -1 & \lambda-5 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$$ 1

$-4(3\lambda-12)+6(21)-2(8\lambda-39) = 0 \Rightarrow \lambda = 9$ 1½

14. $\vec{a}_1 = \hat{i}+2\hat{j}+\hat{k},\ \vec{b}_1 = \hat{i}-\hat{j}+\hat{k}$

$\vec{a}_2 = 2\hat{i}-\hat{j}-\hat{k},\ \vec{b}_2 = 2\hat{i}+\hat{j}+2\hat{k}$ 1

$$\vec{a}_2-\vec{a}_1 = \hat{i}-3\hat{j}-2\hat{k},\ \vec{b}_1\times\vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -3\hat{i}+3\hat{k}$$ ½ + 1

$|\vec{b}_1\times\vec{b}_2| = 3\sqrt{2}$ ½

$(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1-\vec{b}_2) = -3-6 = -9$

Shortest distance $= \left|\dfrac{-9}{3\sqrt{2}}\right| = \dfrac{3\sqrt{2}}{2}$ 1

15. $\tan^{-1}2x+\tan^{-1}3x = \dfrac{\pi}{4}$

$\tan^{-1}\left(\dfrac{5x}{1-6x^2}\right) = \dfrac{\pi}{4}$ 1

$\dfrac{5x}{1-6x^2} = 1$ 1

$\Rightarrow 6x^2 + 5x - 1 = 0$ 1

$\Rightarrow x = \frac{1}{6}, x = -1$ (rejected) 1

OR

$\sin^{-1}\frac{5}{13} = \tan^{-1}\frac{5}{12}$ 1

$\cos^{-1}\frac{3}{5} = \tan^{-1}\frac{4}{3}$ 1

$\text{R.H.S.} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5} = \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{4}{3}$

$= \tan\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\cdot\frac{4}{3}}\right)$ 1

$= \tan^{-1}\frac{63}{16}$ 1

16. $y = \frac{x\cos^{-1}x}{\sqrt{1-x^2}} - \log\sqrt{1-x^2}$

$\frac{dy}{dx} = \frac{\sqrt{1-x^2}\cdot\left(1\cos^{-1}x - \frac{x}{\sqrt{1-x^2}}\right) - \frac{x\cos^{-1}x(-2x)}{2\sqrt{1-x^2}}}{1-x^2} + \frac{2x}{2(1-x^2)}$ 1+1

$= \frac{\sqrt{1-x^2}\cos^{-1}x - x + \frac{x^2\cos^{-1}x}{1-x^2}}{1-x^2} + \frac{x}{1-x^2}$ 1

$= \frac{(1-x^2)\cos^{-1}x + x^2\cos^{-1}x}{(1-x^2)^{3/2}} = \frac{\cos^{-1}x}{(1-x^2)^{3/2}}$ 1

17. $y = (\sin x)^x + \sin^{-1}\sqrt{x}$

$\Rightarrow y = e^{x \log \sin x} + \sin^{-1}\sqrt{x}$ 1

$\Rightarrow \frac{dy}{dx} = e^{x \log \sin x}[\log \sin x + x \cot x] + \frac{1}{2\sqrt{x}\,\sqrt{1-x}}$ 1½

$\Rightarrow \frac{dy}{dx} = (\sin x)^x\ (\log \sin x + x \cot x) + \frac{1}{2\sqrt{x}\,\sqrt{1-x}}$ 1½

18. $x = a \sec^3 \theta$

$\frac{dx}{d\theta} = 3a \sec^3 \theta \tan \theta$ ½

$y = a \tan^3 \theta$

$\frac{dy}{d\theta} = 3a \tan^2 \theta \sec^2 \theta$ ½

$\frac{dy}{dx} = \frac{3a \tan^2 \theta \sec^2 \theta}{3a \sec^3 \theta \tan \theta} = \sin \theta$ 1

$\frac{d^2y}{dx^2} = \cos \theta \cdot \frac{d\theta}{dx} = \frac{\cos \theta}{3a \sec^3 \theta \tan \theta} = \frac{\cos^4 \theta}{3a \tan \theta}$ 1

$\left.\frac{d^2y}{dx^2}\right]_{\theta = \frac{\pi}{4}} = \frac{1}{12a}$ 1

19. $\int \frac{e^x (x^2+1)}{(x+1)^2}\, dx$

$= \int e^x \left[\frac{(x^2-1)+2}{(x+1)^2}\right] dx$ 1

$= \int e^x \left[\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right] dx$ 1

$= \frac{x-1}{x+1} \cdot e^x - \int \frac{2}{(x+1)^2} e^x \, dx + \int \frac{2}{(x+1)^2} e^x \, dx$ 1

$= \frac{e^x (x-1)}{x+1} + C$ 1

SECTION C

20. (a, b) * (c, d) = (a + c, b + d) = (c + a, d + b) = (c, d) * (a, b) $\therefore$ * is commutative 1½

[(a, b) * (c, d)] * (e, f) = (a + c, b + d) * (e, f)

= (a + c + e, b + d + f) = (a, b) * (c + e, d + f) 1

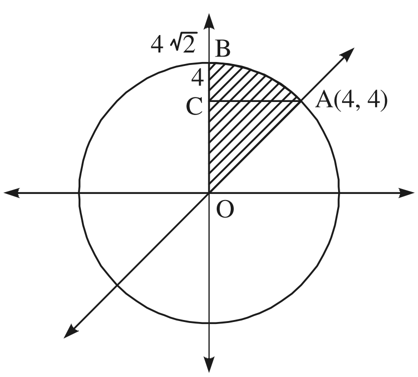
= (a, b) * [(c, d) * (e, f) $\therefore$ * is associate 1½

Let (e, e′) be the identity

(a, b) * (e, e′) = (a, b) ⇒ (a + e, b + e′) = (a, b) ⇒ e = 0, e′ = 0

⇒ Identity element is (0, 0) 2

21.



$x^2 + y^2 = 32$; $y = x$ point of intersection is y = 4 ½

Correct figure 1

Required Area = $\int_0^4 y \, dy + \int_4^{4\sqrt{2}} \sqrt{32 - y^2} \, dy$ 1½

$= \left[\frac{y^2}{2}\right]_0^4 + \left[\frac{y}{2}\sqrt{32-y^2} + 16 \sin^{-1} \frac{y}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$ 1½

$= 8 + \left(0 + 16 \cdot \frac{\pi}{2}\right) - \left(8 + 16 \cdot \frac{\pi}{2}\right) = 4\pi$ 1½

22. $x\frac{dy}{dx} + y - x + xy \cot x = 0 \Rightarrow \frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right) y = 1$ ½

I.F. = $e^{\int \left(\frac{1}{x} + \cot x\right) dx} = x \sin x$ 1

Solution: $y \cdot x \sin x = \int 1 \cdot x \sin x \, dx$ 1½

$\Rightarrow yx \sin x = -x \cos x + \sin x + C$ 1

when

$x = \frac{\pi}{2}$, $y = 0$, we have $C = -1$ 1

$yx \sin x + x \cos x - \sin x = 1$ 1

OR

$x^2 dy + (xy + y^2)dx = 0 \Rightarrow \frac{dy}{dx} = \frac{-(xy + y^2)}{x^2}$ 1

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

$\Rightarrow v + x\frac{dv}{dx} = -(v + v^2) \Rightarrow \frac{dv}{v^2 + 2v} = -\frac{dx}{dx}$

$\Rightarrow \int \frac{dv}{(v+1)^2 - (1)^2} - \int \frac{dx}{x} \Rightarrow \frac{1}{2}\log\frac{v}{v+2} = -\log x + \log C$ 1

$\Rightarrow \frac{C}{x} = \sqrt{\frac{y}{y+x}}$ 1

If $x = 1$, $y = 1$, then $C = \frac{1}{\sqrt{3}}$ 1

$\Rightarrow \frac{1}{\sqrt{3}\,x} = \sqrt{\frac{y}{y+x}}$ 1

23. Plane passing through the intersection of given planes:

$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$ 1

$(1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z + (-1 - 5\lambda) = 0$ 1½

Now $(1 + 2\lambda)\,1 + (1 + 3\lambda)\,(-1) + (1 + 4\lambda)1 = 0$ 1½

$\Rightarrow \lambda = -\frac{1}{3}$ 1

Equation of required plane is

$\Rightarrow x - z + 2 = 0$ 1

24. E_1: First bag is selected.

E_2: Second bag is selected.

A: both balls are red. } 1

$P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}, P\left(\frac{A}{E_1}\right) = \frac{12}{56}, P\left(\frac{A}{E_2}\right) = \frac{2}{56}$ ½ + ½ + 1 + 1

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right)} = \frac{\frac{1}{2}\times\frac{12}{56}}{\frac{1}{2}\times\frac{12}{56} + \frac{1}{2}\cdot\frac{2}{56}} = \frac{6}{7}$$ ½ + 1½

25. Let x and y be the number of takes. Then

Maximise:

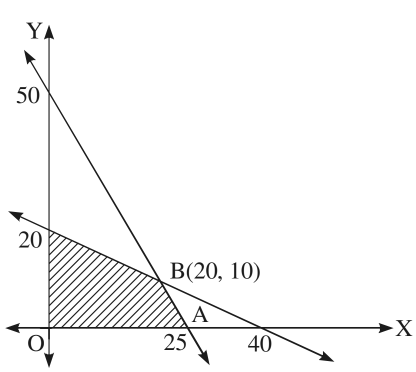
$z = x + y$ 1

Subject to:

$200x + 100y \leq 5000$

$25x + 50y \leq 1000$

$x \geq 0, y \geq 0$ } 2



Correct figure 2

at (20, 10), z = 20 + 10 = 30 is maximum.

at (25, 0), z = 25 + 0 = 25

at (0, 20), z = 20 } 1

26. $l \times b \times 3 = 75 \Rightarrow l \times b = 25$ 1

Let C be the cost. Then

$C = 100\ (l \times b) + 100\ h(b + l)$ 1

$C = 100\left(l \times \frac{25}{l}\right) + 300\left(\frac{25}{l} + l\right)$ 1

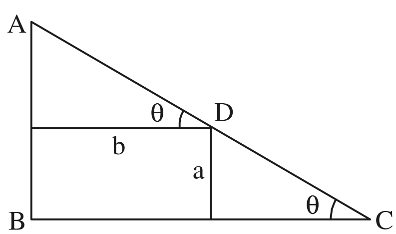
$\frac{dC}{dl} = 0 + 300\left(\frac{-25}{l^2} + 1\right)$

$\frac{dC}{dl} = 0 \Rightarrow l = 5$ 1

$\frac{d^2C}{dl^2} > 0 \Rightarrow$ C is maximum when $l = 5 \Rightarrow b = 5$ 1

C = 100 (25) + 300(10)) = Rs. 5500 1

OR



Correct figure 1

AD = b sec θ, DC = a cosec θ 1

L = AC = b sec θ + a cosec θ 1

$\frac{dL}{d\theta} = b \sec\theta \tan\theta - a\text{cosec}\,\theta \cot\theta$ 1

$\frac{dL}{d\theta} = 0 \Rightarrow \tan^3\theta = \frac{a}{b}$ 1

$\frac{d^2L}{d\theta^2} > 0 \Rightarrow$ minima

$L = \frac{b \cdot \sqrt{a^{2/3} + b^{2/3}}}{b^{1/3}} + \frac{a\sqrt{a^{2/3} + b^{2/3}}}{a^{1/3}}$

$\Rightarrow L = (a^{2/3} + b^{2/3})^{2/3}$ 1

QUESTION PAPER CODE 65/1/2

EXPECTED ANSWER/VALUE POINTS

SECTION A

Marks

1. $1 \cdot y + x\dfrac{dy}{dx} = -c\sin x \Rightarrow x\dfrac{dy}{dx} + y + xy\tan x = 0$ ½ + ½

2. order = 2, degree = 3, sum = 2 + 3 = 5 ½ + ½

3. $\hat{a} = \dfrac{1}{\sqrt{5}}\hat{i} - \dfrac{2}{\sqrt{5}}\hat{j}$ then $7\hat{a} = \dfrac{7}{\sqrt{5}}\hat{i} - \dfrac{14}{\sqrt{5}}\hat{j}$ ½ + ½

4. $(\sqrt{2}\vec{a} - \vec{b}) \cdot (\sqrt{2}\vec{a} - \vec{b}) = 1 \Rightarrow \theta = \dfrac{\pi}{4}$ ½ + ½

5. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$ ½ + ½

6. $AB = \begin{bmatrix} -1 & 5 \\ 4 & 8 \end{bmatrix} \Rightarrow |AB| = -28$ ½ + ½

SECTION B

7. $y = \dfrac{x\cos^{-1}x}{\sqrt{1-x^2}} - \log\sqrt{1-x^2}$

$$\frac{dy}{dx} = \frac{\sqrt{1-x^2}\cdot\left(1\cos^{-1}x - \dfrac{x}{\sqrt{1-x^2}}\right) - \dfrac{x\cos^{-1}x(-2x)}{2\sqrt{1-x^2}}}{1-x^2} + \frac{2x}{2(1-x^2)}$$ 1+1

$$= \frac{\sqrt{1-x^2}\,\cos^{-1}x - x + \dfrac{x^2\cos^{-1}x}{1-x^2}}{1-x^2} + \frac{x}{1-x^2}$$ 1

$$= \frac{(1-x^2)\cos^{-1}x + x^2\cos^{-1}x}{(1-x^2)^{3/2}} = \frac{\cos^{-1}x}{(1-x^2)^{3/2}}$$ 1

8. $y = (\sin x)^x + \sin^{-1}\sqrt{x}$

$\Rightarrow y = e^{x \log \sin x} + \sin^{-1}\sqrt{x}$ 1

$\Rightarrow \frac{dy}{dx} = e^{x \log \sin x}[\log \sin x + x \cot x] + \frac{1}{2\sqrt{x}\sqrt{1-x}}$ 1½

$\Rightarrow \frac{dy}{dx} = (\sin x)^x (\log \sin x + x \cot x) + \frac{1}{2\sqrt{x}\sqrt{1-x}}$ 1½

9. $x = a \sec^3 \theta$

$\frac{dx}{d\theta} = 3a \sec^3 \theta \tan \theta$ ½

$y = a \tan^3 \theta$

$\frac{dy}{d\theta} = 3a \tan^2 \theta \sec^2 \theta$ ½

$\frac{dy}{dx} = \frac{3a \tan^2 \theta \sec^2 \theta}{3a \sec^3 \theta \tan \theta} = \sin \theta$ 1

$\frac{d^2y}{dx^2} = \cos \theta \cdot \frac{d\theta}{dx} = \frac{\cos \theta}{3a \sec^3 \theta \tan \theta} = \frac{\cos^4 \theta}{3a \tan \theta}$ 1

$\left.\frac{d^2y}{dx^2}\right]_{\theta = \frac{\pi}{4}} = \frac{1}{12a}$ 1

10. $\int \frac{e^x (x^2 + 1)}{(x+1)^2} dx$

$= \int e^x \left[\frac{(x^2 - 1) + 2}{(x+1)^2}\right] dx$ 1

$= \int e^x \left[\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right] dx$ 1

$$= \frac{x-1}{x+1} \cdot e^x - \int \frac{2}{(x+1)^2} e^x \, dx + \int \frac{2}{(x+1)^2} e^x \, dx$$ 1

$$= \frac{e^x (x-1)}{x+1} + C$$ 1

11. System of equation is

3x + y + 2z = 1100, x + 2y + 3z = 1400, x + y + z = 600 1½

(i) Matrix equation is

$$\begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1100 \\ 1400 \\ 600 \end{bmatrix}$$ 1

(ii) $|A| = -3 \neq 0$, system of equations can be solved. ½

(iii) Any one value with reason. 1

12. $$\begin{bmatrix} 2x & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2x-9 & 4x \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = 0$$ 1

$$[2x^2 - 9x + 12x] = [0] \Rightarrow 2x^2 + 3x = 0, \; x = 0 \text{ or } \frac{-3}{2}$$ 1+1+1

13. $$\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ 4a+10 & 2 & 0 \end{vmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$ 1+1

= 4a + 8 – 4a – 10 = –2. 1+1

14. $I = \int_0^{\pi/2} \frac{1}{1+\sqrt{\tan x}} dx$

$I = \int_0^{\pi/2} \frac{1}{1+\sqrt{\tan(\pi/2 - x)}} dx = \int_0^{\pi/2} \frac{1}{1+\sqrt{\cot x}} dx = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}} dx$ 1½

$\Rightarrow 2I = \int_0^{\pi/2} \frac{1+\sqrt{\tan x}}{1+\sqrt{\tan x}} dx = \int_0^{\pi/2} 1 \cdot dx = \frac{\pi}{2}$ 1½

$\Rightarrow I = \frac{\pi}{4}$ 1

15. $\frac{x}{(x-1)^2 (x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$ ½

$A = \frac{2}{9}, B = \frac{1}{3}, C = -\frac{2}{9}$ 1½

$\int \frac{x}{(x-1)^2 (x+2)} dx = \int \frac{2}{9(x-1)} dx + \int \frac{1}{3(x-1)^2} dx - \int \frac{2}{9(x+2)} dx$ 1½

$= \frac{2}{9} \log|x-1| - \frac{1}{3(x-1)} - \frac{2}{9} \log|x+2| + C$ 1½

16. Let X be the number of defective bulbs. Then

X = 0, 1, 2 1

$P(X=0) = \frac{10_{C_2}}{15_{C_2}} = \frac{3}{7}$, $P(X=1) = \frac{10_{C_1} \cdot 5_{C_1}}{15_{C_2}} = \frac{10}{21}$ 1+1

$P(X=2) = \frac{5_{C_2}}{15_{C_2}} = \frac{2}{21}$ 1

X	0	1	2
P(X)	$\frac{3}{7}$	$\frac{10}{21}$	$\frac{2}{21}$

OR

E_1: Problem is solved by A.

E_2: Problem is solved by B.

$P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{3}, P(\bar{E}_1) = \frac{1}{2}, P(\bar{E}_2) = \frac{2}{3}$ 1

$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2) = \frac{1}{6}$

P(problem is solved) = $1 - P(\bar{E}_1) \cdot P(\bar{E}_2) = 1 - \frac{1}{2} \cdot \frac{2}{3} = \frac{2}{3}$ 1½

P(one of them is solved) = $P(E_1)P(\bar{E}_2) + P(\bar{E}_1)P(E_2)$

$= \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{2}$ 1½

17. $\left.\begin{aligned} \overrightarrow{AB} &= -4\hat{i} - 6\hat{j} - 2\hat{k} \\ \overrightarrow{AC} &= -\hat{i} + (\lambda - 5)\,\hat{j} + 3\hat{k} \\ \overrightarrow{AD} &= -8\hat{i} - \hat{j} + 3\hat{k} \end{aligned}\right\}$ 1½

$\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) = \begin{vmatrix} -4 & -6 & -2 \\ -1 & \lambda - 5 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$ 1

$-4(3\lambda - 12) + 6(21) - 2(8\lambda - 39) = 0 \Rightarrow \lambda = 9$ 1½

18. $\left.\begin{aligned} \vec{a}_1 &= \hat{i} + 2\hat{j} + \hat{k}, \vec{b}_1 = \hat{i} - \hat{j} + \hat{k} \\ \vec{a}_2 &= 2\hat{i} - \hat{j} - \hat{k}, \vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k} \end{aligned}\right\}$ 1

$\vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}, \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -3\hat{i} + 3\hat{k}$ ½ + 1

$|\vec{b}_1 \times \vec{b}_2| = 3\sqrt{2}$ ½

$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 - \vec{b}_2) = -3 - 6 = -9$

Shortest distance $= \left|\frac{-9}{3\sqrt{2}}\right| = \frac{3\sqrt{2}}{2}$ 1

19. $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \frac{\pi}{4}$ 1

$\frac{5x}{1-6x^2} = 1$ 1

$\Rightarrow 6x^2 + 5x - 1 = 0$ 1

$\Rightarrow x = \frac{1}{6}, x = -1$ (rejected) 1

OR

$\sin^{-1}\frac{5}{13} = \tan^{-1}\frac{5}{12}$ 1

$\cos^{-1}\frac{3}{5} = \tan^{-1}\frac{4}{3}$ 1

R.H.S. $= \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5} = \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{4}{3}$

$= \tan\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\cdot\frac{4}{3}}\right)$ 1

$= \tan^{-1}\frac{63}{16}$ 1

SECTION C

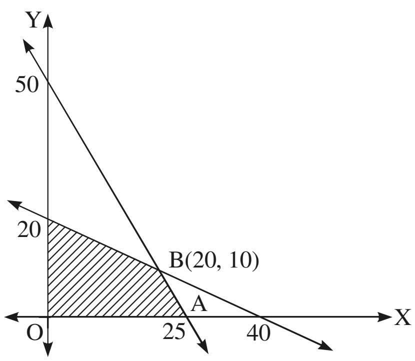
20. Let x and y be the number of takes. Then

Maximise:

$z = x + y$ 1

Subject to:

$$\left.\begin{aligned} 200x + 100y &\le 5000 \\ 25x + 50y &\le 1000 \\ x \ge 0,\ y &\ge 0 \end{aligned}\right\}$$ 2



Correct figure 2

$$\left.\begin{aligned} &\text{at } (20, 10),\ z = 20 + 10 = 30 \text{ is maximum.} \\ &\text{at } (25, 0),\ z = 25 + 0 = 25 \\ &\text{at } (0, 20),\ z = 20 \end{aligned}\right\}$$ 1

21. $l \times b \times 3 = 75 \Rightarrow l \times b = 25$ 1

Let C be the cost. Then

$C = 100\,(l \times b) + 100\,h(b + l)$ 1

$$C = 100\left(l \times \frac{25}{l}\right) + 300\left(\frac{25}{l} + l\right)$$ 1

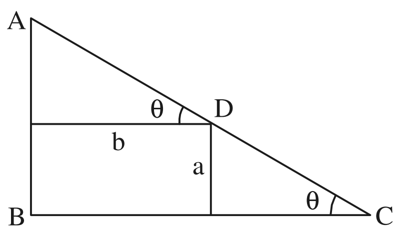
$$\frac{dC}{dl} = 0 + 300\left(\frac{-25}{l^2} + 1\right)$$

$$\frac{dC}{dl} = 0 \Rightarrow l = 5$$ 1

$$\frac{d^2C}{dl^2} > 0 \Rightarrow \text{C is maximum when } l = 5 \Rightarrow b = 5$$ 1

$C = 100\,(25) + 300(10)) = \text{Rs. } 5500$ 1

OR



Correct figure 1

$AD = b \sec\theta,\ DC = a \operatorname{cosec}\theta$ 1

$L = AC = b\sec\theta + a \operatorname{cosec}\theta$ 1

$\frac{dL}{d\theta} = b\sec\theta\tan\theta - a\operatorname{cosec}\theta\cot\theta$ 1

$\frac{dL}{d\theta} = 0 \Rightarrow \tan^3\theta = \frac{a}{b}$ 1

$\frac{d^2L}{d\theta^2} > 0 \Rightarrow$ minima

$L = \frac{b\cdot\sqrt{a^{2/3}+b^{2/3}}}{b^{1/3}} + \frac{a\sqrt{a^{2/3}+b^{2/3}}}{a^{1/3}}$

$\Rightarrow L = (a^{2/3}+b^{2/3})^{2/3}$ 1

22. (a, b) * (c, d) = (a + c, b + d) = (c + a, d + b) = (c, d) * (a, b) $\therefore$ * is commutative 1½

[(a, b) * (c, d)] * (e, f) = (a + c, b + d) * (e, f)

= (a + c + e, b + d + f) = (a, b) * (c + e, d + f) 1

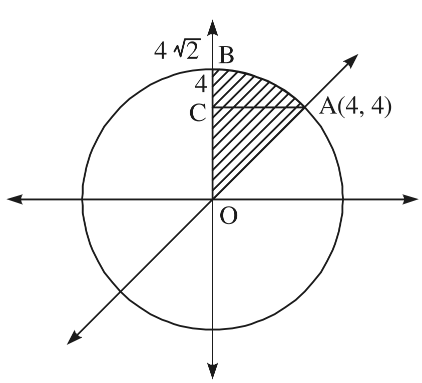
= (a, b) * [(c, d) * (e, f) $\therefore$ * is associate 1½

Let (e, e′) be the identity

(a, b) * (e, e′) = (a, b) $\Rightarrow$ (a + e, b + e′) = (a, b) $\Rightarrow$ e = 0, e′ = 0

$\Rightarrow$ Identity element is (0, 0) 2

23.



$x^2 + y^2 = 32;\ y = x$ point of intersection is y = 4 ½

Correct figure 1

Required Area $= \int_0^4 y\,dy + \int_4^{4\sqrt{2}} \sqrt{32 - y^2}\,dy$ 1½

$$= \left[\frac{y^2}{2}\right]_0^4 + \left[\frac{y}{2}\sqrt{32-y^2} + 16\sin^{-1}\frac{y}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$ 1½

$$= 8 + \left(0 + 16\cdot\frac{\pi}{2}\right) - \left(8 + 16\cdot\frac{\pi}{2}\right) = 4\pi$$ 1½

24. $x\frac{dy}{dx} + y - x + xy\cot x = 0 \Rightarrow \frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right)y = 1$ ½

I.F. = $e^{\int\left(\frac{1}{x}+\cot x\right)dx} = x \sin x$ 1

Solution: $y \cdot x \sin x = \int 1 \cdot x \sin x \, dx$ 1½

$\Rightarrow yx \sin x = -x \cos x + \sin x + C$ 1

when

$x = \frac{\pi}{2}$, $y = 0$, we have $C = -1$ 1

$yx \sin x + x \cos x - \sin x = 1$ 1

OR

$x^2 dy + (xy + y^2)dx = 0 \Rightarrow \frac{dy}{dx} = \frac{-(xy+y^2)}{x^2}$ 1

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

$\Rightarrow v + x\frac{dv}{dx} = -(v + v^2) \Rightarrow \frac{dv}{v^2 + 2v} = -\frac{dx}{dx}$

$\Rightarrow \int \frac{dv}{(v+1)^2 - (1)^2} - \int \frac{dx}{x} \Rightarrow \frac{1}{2}\log\frac{v}{v+2} = -\log x + \log C$ 1

$\Rightarrow \frac{C}{x} = \sqrt{\frac{y}{y+x}}$ 1

If x = 1, y = 1, then $C = \frac{1}{\sqrt{3}}$ 1

$\Rightarrow \frac{1}{\sqrt{3}\,x} = \sqrt{\frac{y}{y+x}}$ 1

25. Plane passing through the intersection of given planes:

$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$ 1

$(1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z + (-1 - 5\lambda) = 0$ 1½

Now $(1 + 2\lambda)\,1 + (1 + 3\lambda)\,(-1) + (1 + 4\lambda)1 = 0$ 1½

$\Rightarrow \lambda = -\frac{1}{3}$ 1

Equation of required plane is

$\Rightarrow x - z + 2 = 0$ 1

26. E_1: First bag is selected.

E_2: Second bag is selected. } 1

A: both balls are red.

$P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}, P\left(\frac{A}{E_1}\right) = \frac{12}{56}, P\left(\frac{A}{E_2}\right) = \frac{2}{56}$ ½ + ½ + 1 + 1

$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right)} = \frac{\frac{1}{2}\times\frac{12}{56}}{\frac{1}{2}\times\frac{12}{56} + \frac{1}{2}\cdot\frac{2}{56}} = \frac{6}{7}$ ½ + 1½

QUESTION PAPER CODE 65/1/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

Marks

1. $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1 \Rightarrow \sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$ ½ + ½

2. $AB=\begin{bmatrix}-1 & 5\\ 4 & 8\end{bmatrix} \Rightarrow |AB|=-28$ ½ + ½

3. $1\cdot y+x\dfrac{dy}{dx}=-c\sin x \Rightarrow x\dfrac{dy}{dx}+y+xy\tan x=0$ ½ + ½

4. order = 2, degree = 3, sum = 2 + 3 = 5 ½ + ½

5. $\hat{a}=\dfrac{1}{\sqrt{5}}\hat{i}-\dfrac{2}{\sqrt{5}}\hat{j}$ then $7\hat{a}=\dfrac{7}{\sqrt{5}}\hat{i}-\dfrac{14}{\sqrt{5}}\hat{j}$ ½ + ½

6. $(\sqrt{2}\vec{a}-\vec{b})\cdot(\sqrt{2}\vec{a}-\vec{b})=1 \Rightarrow \theta=\dfrac{\pi}{4}$ ½ + ½

SECTION B

7. $\dfrac{x}{(x-1)^2\,(x+2)}=\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x+2}$ ½

$A=\dfrac{2}{9}, B=\dfrac{1}{3}, C=-\dfrac{2}{9}$ 1½

$\int\dfrac{x}{(x-1)^2\,(x+2)}dx=\int\dfrac{2}{9(x-1)}dx+\int\dfrac{1}{3(x-1)^2}dx-\int\dfrac{2}{9(x+2)}dx$ 1½

$=\dfrac{2}{9}\log|x-1|-\dfrac{1}{3(x-1)}-\dfrac{2}{9}\log|x+2|+C$ 1½

8. Let X be the number of defective bulbs. Then

X = 0, 1, 2 1

$$P(X=0) = \frac{{}^{10}C_2}{{}^{15}C_2} = \frac{3}{7},\ P(X=1) = \frac{{}^{10}C_1 \cdot {}^{5}C_1}{{}^{15}C_2} = \frac{10}{21}$$ 1+1

$$P(X=2) = \frac{{}^{5}C_2}{{}^{15}C_2} = \frac{2}{21}$$ 1

X	0	1	2
P(X)	$\frac{3}{7}$	$\frac{10}{21}$	$\frac{2}{21}$

OR

E_1: Problem is solved by A.

E_2: Problem is solved by B.

$$P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{3}, P(\bar{E}_1) = \frac{1}{2}, P(\bar{E}_2) = \frac{2}{3}$$ 1

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2) = \frac{1}{6}$$

$$\text{P(problem is solved)} = 1 - P(\bar{E}_1) \cdot P(\bar{E}_2) = 1 - \frac{1}{2} \cdot \frac{2}{3} = \frac{2}{3}$$ 1½

$$\text{P(one of them is solved)} = P(E_1)P(\bar{E}_2) + P(\bar{E}_1)P(E_2)$$

$$= \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{2}$$ 1½

9. $$\left.\begin{aligned} \overrightarrow{AB} &= -4\hat{i} - 6\hat{j} - 2\hat{k} \\ \overrightarrow{AC} &= -\hat{i} + (\lambda - 5)\hat{j} + 3\hat{k} \\ \overrightarrow{AD} &= -8\hat{i} - \hat{j} + 3\hat{k} \end{aligned}\right\}$$ 1½

$$\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) = \begin{vmatrix} -4 & -6 & -2 \\ -1 & \lambda - 5 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$$ 1

$-4(3\lambda-12)+6(21)-2(8\lambda-39)=0 \Rightarrow \lambda=9$ 1½

10. $\left.\begin{array}{l}\vec{a}_1 = \hat{i}+2\hat{j}+\hat{k}, \vec{b}_1=\hat{i}-\hat{j}+\hat{k} \\ \vec{a}_2 = 2\hat{i}-\hat{j}-\hat{k}, \vec{b}_2=2\hat{i}+\hat{j}+2\hat{k}\end{array}\right\}$ 1

$\vec{a}_2-\vec{a}_1 = \hat{i}-3\hat{j}-2\hat{k}, \vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 1\\ 2 & 1 & 1\end{vmatrix}=-3\hat{i}+3\hat{k}$ ½ + 1

$|\vec{b}_1\times\vec{b}_2| = 3\sqrt{2}$ ½

$(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1-\vec{b}_2) = -3-6=-9$

Shortest distance = $\left|\frac{-9}{3\sqrt{2}}\right|=\frac{3\sqrt{2}}{2}$ 1

11. $\tan^{-1}2x+\tan^{-1}3x = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \frac{\pi}{4}$ 1

$\frac{5x}{1-6x^2} = 1$ 1

$\Rightarrow 6x^2+5x-1 = 0$ 1

$\Rightarrow x=\frac{1}{6}, x=-1$ (rejected) 1

OR

$\sin^{-1}\frac{5}{13} = \tan^{-1}\frac{5}{12}$ 1

$\cos^{-1}\frac{3}{5} = \tan^{-1}\frac{4}{3}$ 1

R.H.S. $= \sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}=\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{4}{3}$

$= \tan\left(\dfrac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\cdot\frac{4}{3}}\right)$ 1

$= \tan^{-1}\frac{63}{16}$ 1

12. $y=\dfrac{x\cos^{-1}x}{\sqrt{1-x^2}}-\log\sqrt{1-x^2}$

$\dfrac{dy}{dx}=\dfrac{\sqrt{1-x^2}\cdot\left(1\cos^{-1}x-\frac{x}{\sqrt{1-x^2}}\right)-\frac{x\cos^{-1}x(-2x)}{2\sqrt{1-x^2}}}{1-x^2}+\dfrac{2x}{2(1-x^2)}$ 1+1

$=\dfrac{\sqrt{1-x^2}\ \cos^{-1}x-x+\frac{x^2\cos^{-1}x}{1-x^2}}{1-x^2}+\dfrac{x}{1-x^2}$ 1

$=\dfrac{(1-x^2)\cos^{-1}x+x^2\cos^{-1}x}{(1-x^2)^{3/2}}=\dfrac{\cos^{-1}x}{(1-x^2)^{3/2}}$ 1

13. $y=(\sin x)^x+\sin^{-1}\sqrt{x}$

$\Rightarrow y=e^{x\log\sin x}+\sin^{-1}\sqrt{x}$ 1

$\Rightarrow \dfrac{dy}{dx}=e^{x\log\sin x}[\log\sin x+x\cot x]+\dfrac{1}{2\sqrt{x}\ \sqrt{1-x}}$ 1½

$\Rightarrow \dfrac{dy}{dx}=(\sin x)^x\ (\log\sin x+x\cot x)+\dfrac{1}{2\sqrt{x}\ \sqrt{1-x}}$ 1½

14. $x=a\sec^3\theta$

$\dfrac{dx}{d\theta}=3a\sec^3\theta\tan\theta$ ½

$y = a\tan^3\theta$

$\frac{dy}{d\theta} = 3a\tan^2\theta\sec^2\theta$ ½

$\frac{dy}{dx} = \frac{3a\tan^2\theta\sec^2\theta}{3a\sec^3\theta\tan\theta} = \sin\theta$ 1

$\frac{d^2y}{dx^2} = \cos\theta\cdot\frac{d\theta}{dx} = \frac{\cos\theta}{3a\sec^3\theta\tan\theta} = \frac{\cos^4\theta}{3a\tan\theta}$ 1

$\left.\frac{d^2y}{dx^2}\right]_{\theta=\frac{\pi}{4}} = \frac{1}{12a}$ 1

15. $\int\frac{e^x(x^2+1)}{(x+1)^2}dx$

$= \int e^x\left[\frac{(x^2-1)+2}{(x+1)^2}\right]dx$ 1

$= \int e^x\left[\frac{x-1}{x+1}+\frac{2}{(x+1)^2}\right]dx$ 1

$= \frac{x-1}{x+1}\cdot e^x - \int\frac{2}{(x+1)^2}e^x\,dx + \int\frac{2}{(x+1)^2}e^x\,dx$ 1

$= \frac{e^x(x-1)}{x+1} + C$ 1

16. System of equation is

3x + y + 2z = 1100, x + 2y + 3z = 1400, x + y + z = 600 1½

(i) Matrix equation is

$$\begin{bmatrix}3 & 1 & 2\\1 & 2 & 3\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1100\\1400\\600\end{bmatrix}$$ 1

(ii) |A| = –3 ≠ 0, system of equations can be solved. ½

(iii) Any one value with reason. 1

17. $\begin{bmatrix} 2x & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = 0$

$\begin{bmatrix} 2x-9 & 4x \end{bmatrix} \begin{bmatrix} x \\ 3 \end{bmatrix} = 0$ 1

$[2x^2 - 9x + 12x] = [0] \Rightarrow 2x^2 + 3x = 0,\ x = 0 \text{ or } \frac{-3}{2}$ 1+1+1

18. $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$

$= \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ 4a+10 & 2 & 0 \end{vmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$ 1+1

= 4a + 8 – 4a – 10 = –2. 1+1

19. $I = \int_0^{\pi/2} \frac{1}{1+\sqrt{\tan x}}\,dx$

$I = \int_0^{\pi/2} \frac{1}{1+\sqrt{\tan(\pi/2 - x)}}\,dx = \int_0^{\pi/2} \frac{1}{1+\sqrt{\cot x}}\,dx = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\,dx$ 1½

$\Rightarrow 2I = \int_0^{\pi/2} \frac{1+\sqrt{\tan x}}{1+\sqrt{\tan x}}\,dx = \int_0^{\pi/2} 1 \cdot dx = \frac{\pi}{2}$ 1½

$\Rightarrow I = \frac{\pi}{4}$ 1

SECTION C

20. Plane passing through the intersection of given planes:

$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$ 1

$(1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z + (-1 - 5\lambda) = 0$ 1½

Now $(1 + 2\lambda)\,1 + (1 + 3\lambda)\,(-1) + (1 + 4\lambda)1 = 0$ 1½

$\Rightarrow \lambda = -\frac{1}{3}$ 1

Equation of required plane is

$\Rightarrow x - z + 2 = 0$ 1

21. E_1: First bag is selected.

E_2: Second bag is selected.

A: both balls are red. } 1

$P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}, P\left(\frac{A}{E_1}\right) = \frac{12}{56}, P\left(\frac{A}{E_2}\right) = \frac{2}{56}$ ½ + ½ + 1 + 1

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right)} = \frac{\frac{1}{2}\times\frac{12}{56}}{\frac{1}{2}\times\frac{12}{56} + \frac{1}{2}\cdot\frac{2}{56}} = \frac{6}{7}$$ ½ + 1½

22. Let x and y be the number of takes. Then

Maximise:

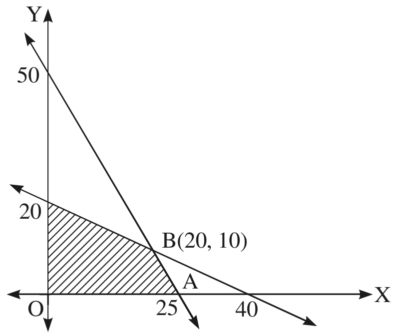
$z = x + y$ 1

Subject to:

$200\,x + 100y \le 5000$

$25x + 50y \le 1000$

$x \ge 0, y \ge 0$ } 2



Correct figure 2

at (20, 10), z = 20 + 10 = 30 is maximum.

at (25, 0), z = 25 + 0 = 25 1

at (0, 20), z = 20

23. $l \times b \times 3 = 75 \Rightarrow l \times b = 25$ 1

Let C be the cost. Then

$C = 100\,(l \times b) + 100\,h(b + l)$ 1

$C = 100\left(l \times \frac{25}{l}\right) + 300\left(\frac{25}{l} + l\right)$ 1

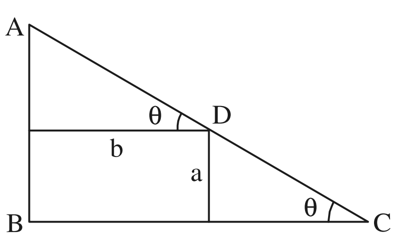
$\frac{dC}{dl} = 0 + 300\left(\frac{-25}{l^2} + 1\right)$

$\frac{dC}{dl} = 0 \Rightarrow l = 5$ 1

$\frac{d^2C}{dl^2} > 0 \Rightarrow$ C is maximum when $l = 5 \Rightarrow b = 5$ 1

C = 100 (25) + 300(10)) = Rs. 5500 1

OR



Correct figure 1

$AD = b \sec\theta$, $DC = a \operatorname{cosec}\theta$ 1

$L = AC = b \sec\theta + a \operatorname{cosec}\theta$ 1

$\frac{dL}{d\theta} = b \sec\theta \tan\theta - a\operatorname{cosec}\theta \cot\theta$ 1

$\frac{dL}{d\theta} = 0 \Rightarrow \tan^3\theta = \frac{a}{b}$ 1

$$\frac{d^2L}{d\theta^2} > 0 \Rightarrow \text{minima}$$

$$L = \frac{b\cdot\sqrt{a^{2/3}+b^{2/3}}}{b^{1/3}} + \frac{a\sqrt{a^{2/3}+b^{2/3}}}{a^{1/3}}$$

$$\Rightarrow L = (a^{2/3}+b^{2/3})^{2/3}$$ 1

24. (a, b) * (c, d) = (a + c, b + d) = (c + a, d + b) = (c, d) * (a, b) ∴ * is commutative 1½

[(a, b) * (c, d)] * (e, f) = (a + c, b + d) * (e, f)

= (a + c + e, b + d + f) = (a, b) * (c + e, d + f) 1

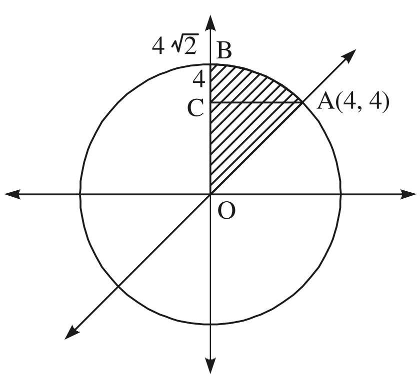
= (a, b) * [(c, d) * (e, f) ∴ * is associate 1½

Let (e, e′) be the identity

(a, b) * (e, e′) = (a, b) ⇒ (a + e, b + e′) = (a, b) ⇒ e = 0, e′ = 0

⇒ Identity element is (0, 0) 2

25.



$x^2 + y^2 = 32$; $y = x$ point of intersection is $y = 4$ ½

Correct figure 1

$$\text{Required Area} = \int_0^4 y\,dy + \int_4^{4\sqrt{2}} \sqrt{32-y^2}\,dy$$ 1½

$$= \left[\frac{y^2}{2}\right]_0^4 + \left[\frac{y}{2}\sqrt{32-y^2} + 16\sin^{-1}\frac{y}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$ 1½

$$= 8 + \left(0 + 16\cdot\frac{\pi}{2}\right) - \left(8 + 16\cdot\frac{\pi}{2}\right) = 4\pi$$ 1½

26. $$x\frac{dy}{dx} + y - x + xy\cot x = 0 \Rightarrow \frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right)y = 1$$ ½

$$\text{I.F.} = e^{\int\left(\frac{1}{x}+\cot x\right)dx} = x\sin x$$ 1

Solution: $y \cdot x \sin x = \int 1 \cdot x \sin x \, dx$ 1½

$\Rightarrow yx \sin x = -x \cos x + \sin x + C$ 1

when

$x = \frac{\pi}{2}, y = 0$, we have $C = -1$ 1

$yx \sin x + x \cos x - \sin x = 1$ 1

OR

$x^2 dy + (xy + y^2) dx = 0 \Rightarrow \frac{dy}{dx} = \frac{-(xy+y^2)}{x^2}$ 1

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

$\Rightarrow v + x\frac{dv}{dx} = -(v+v^2) \Rightarrow \frac{dv}{v^2+2v} = -\frac{dx}{dx}$

$\Rightarrow \int \frac{dv}{(v+1)^2 - (1)^2} - \int \frac{dx}{x} \Rightarrow \frac{1}{2}\log\frac{v}{v+2} = -\log x + \log C$ 1

$\Rightarrow \frac{C}{x} = \sqrt{\frac{y}{y+x}}$ 1

If $x = 1, y = 1$, then $C = \frac{1}{\sqrt{3}}$ 1

$\Rightarrow \frac{1}{\sqrt{3}\,x} = \sqrt{\frac{y}{y+x}}$ 1