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Senior School Certificate Examination

July — 2015 (Comptt.)

Marking Scheme — Mathematics 65(B)

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question (s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65(B)

EXPECTED ANSWER/VALUE POINTS

SECTION A

Marks

1. Position vector of A $= \frac{3\vec{a} + 8\vec{b}}{5}$ ½

   Position vector of B $= \frac{3\vec{a} + 18\vec{b}}{10}$ ½

2. $\vec{a} \cdot \vec{b} = 0$ ½

   $2 - 2\lambda + 3 = 10 \Rightarrow \lambda = \frac{5}{2}$ ½

3. $3(x - 3) + 1\,(y - 1) + 4\,(z - 4) = 0$ ½

   $3x + y + 4z = 26$ ½

4. $a_{31} = -17$ 1

5. $l = 2$, $m = 1$, $l + m = 3$ ½ + ½

6. $y^2 = 4ax - 4ab \Rightarrow y\frac{dy}{dx} = 2a$ ½

   $\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} = 0$ ½

SECTION B

7. $\begin{bmatrix} 260 & 450 & 260 \\ 350 & 560 & 300 \\ 295 & 375 & 250 \end{bmatrix}\begin{bmatrix} 30 \\ 10 \\ 10 \end{bmatrix} = \begin{bmatrix} 14900 \\ 19100 \\ 15100 \end{bmatrix}$ 2

   Total expenditure = 49100 1

   Any relevant value 1

8. $A^2 - xA + yI = 0$

$$\Rightarrow \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix} - \begin{bmatrix} 4x & 3x \\ 2x & 5x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} = 0$$ 2

$$\begin{bmatrix} 22-4x+y & 27-3x \\ 18-2x & 31-5x+y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$ 1

$$\left.\begin{matrix} 27-3x=0 \Rightarrow x=9 \\ 31-5x+y=0 \Rightarrow y=14 \end{matrix}\right\}$$ ½+½

OR

$$\text{Adj } A = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$ 2

$|A| = 1(-3) - 2(-2) + 4 = 5$ 1

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = |A|I$$ 1

9. $$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$= 2\begin{vmatrix} b+c & a+c & a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}; \; R_1 \to R_1 + R_2 + R_3$$ 1

$$= 2\begin{vmatrix} b+c & a+c & a+b \\ -c & o & -a \\ -b & -a & o \end{vmatrix}; \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$ 1

$$= 2\begin{vmatrix} 0 & c & b \\ -c & 0 & -a \\ -b & -a & 0 \end{vmatrix}, \quad R_1 \to R_1 + R_2 + R_3$$ ½

$$= \frac{2}{bc}\begin{vmatrix} 0 & bc & bc \\ -c & 0 & -ac \\ -b & -ab & 0 \end{vmatrix}, \quad C_2 \to bC_2, C_3 \to cC_3$$ ½

$$= \frac{2}{bc}\begin{vmatrix} 0 & 0 & bc \\ -c & ac & -ac \\ -b & -ab & 0 \end{vmatrix}, \quad C_2 \to C_2 - C_3$$ ½

$$= \frac{2}{bc} \cdot bc(abc + abc) = 4abc$$ ½

10. $\int_{1/4}^{1/2} \frac{1}{\sqrt{x - x^2}} dx$

$$= \int_{1/4}^{1/2} \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}} dx$$ 2

$$= \left[\sin^{-1}(2x-1)\right]_{1/4}^{1/2}$$ 1

$$\sin^{-1} 0 - \sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{6}$$ 1

11. $I = \int\left[\log(\log x) + \frac{1}{(\log x)^2}\right]dx$

$\log x = t \Rightarrow x = e^t \Rightarrow dx = e^t dt$ 1

$$1 = \int\left(\log t + \frac{1}{t^2}\right)e^t \, dt$$ ½

$$= \int\left(\log t + \frac{1}{t}\right)e^t dt + \left(-\frac{1}{t} + \frac{1}{t^2}\right)e^t dt$$ 1

$$= e^t \log t + e^t\left(-\frac{1}{t}\right) + C$$ 1

$$= x \cdot \log(\log x) - \frac{x}{\log x} + C$$ ½

12. $P(B) = \dfrac{6}{216}$ 1

$P(A \cap B) = \dfrac{1}{216}$ 1

$P(A/B) = \dfrac{P(A \cap B)}{P(B)}$ 1

$= \dfrac{\frac{1}{216}}{\frac{6}{216}} = \dfrac{1}{6}$ 1

OR

Let X be the number of aces

X = 0, 1, 2 ½

$P(0) = \dfrac{144}{169}, P(1) = \dfrac{24}{169}, P(2) = \dfrac{1}{169}$ 2½

Mean $= 0 \times \dfrac{144}{169} + 1 \times \dfrac{24}{169} + 2 \times \dfrac{1}{169} = \dfrac{2}{13}$ 1

13. $\vec{a}.(\vec{b}+\vec{c}) = 0, \vec{b}.(\vec{a}+\vec{c}) = 0, \vec{c}.(\vec{a}+\vec{b}) = 0$

$\Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 0$ 1

$|\vec{a}+\vec{b}+\vec{c}|^2 = (\vec{a}+\vec{b}+\vec{c}) \cdot (\vec{a}+\vec{b}+\vec{c})$ ½

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{c}.\vec{a} = 9 + 16 + 25 + 0$ 2

$|\vec{a}+\vec{b}+\vec{c}| = 5\sqrt{2}$ ½

14. $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}, \vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}, \vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}$ ½

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = -3\hat{i} + 3\hat{k}$ 1

$$\text{S.D.} = \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$ 1

$$= \left|\frac{(\hat{i} - 3\hat{j} - 2\hat{k})\cdot(-3\hat{i} + 3\hat{k})}{3\sqrt{2}}\right|$$ 1

$$= \frac{3\sqrt{2}}{2}$$ ½

15. $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7}$

$$= \tan^{-1}\frac{2\left(\frac{1}{2}\right)}{1-\frac{1}{4}} + \tan^{-1}\frac{1}{7}$$ 1

$$= \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{7}$$ 1

$$= \tan^{-1}\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}$$ 1

$$= \tan^{-1}\left(\frac{31}{17}\right)$$ 1

OR

$$\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$$

$$\sin^{-1}\frac{5}{x} = \frac{\pi}{2} - \sin^{-1}\frac{2}{x} = \cos^{-1}\frac{12}{x}$$ 1

$$\sin^{-1}\frac{5}{x} = \sin^{-1}\sqrt{1-\frac{144}{x^2}}$$ 1

$\frac{25}{x^2} = 1 - \frac{144}{x^2}$ 1

$\frac{169}{x^2} = 1$

x = +13, x = –13 (rejected) ½+½

16. $x = a\left(\cos t + \log \tan \frac{1}{2}\right), \frac{dx}{dt} = a\left(-\sin t + \frac{1}{\sin t}\right) = a\frac{\cos^2 t}{\sin t}$ 1

$y = a \sin t, \ \frac{dy}{dt} = a \cos t$ 1

$\frac{dy}{dx} = \frac{a \cos t \sin t}{a \cos^2 t} = \tan t$ 1

$\frac{d^2y}{dx^2} = \sec^2 t \cdot \frac{\sin t}{a \cos^2 t} = \frac{1}{a} \tan t \sec^3 t$ 1

17. $y = (\cos x)^x + (\sin x)^{\frac{1}{x}}$

$y = e^{x \log \cos x} + e^{\frac{1}{x} \log \sin x}$ 1

$\frac{dy}{dx} = e^{x \log \cos x}\left[\log \cos x - x \tan x\right] + e^{\frac{1}{x} \log \sin x}\left[-\frac{1}{x^2} \log \sin x + \frac{1}{x} \cot x\right]$ 1+1

$\frac{dy}{dx} = (\cos x)^x [\log \cos x - x \tan x] + \frac{(\sin x)^{1/x}}{x}\left[-\frac{1}{x} \log \sin x + \cot x\right]$ 1

18. $x = a \sin^3 t \Rightarrow \frac{dx}{dt} = 3a \sin^2 t \cos t$ 1

$y = b \cos^3 t \Rightarrow \frac{dy}{dt} = -3b \cos^2 t \sin t$ 1

$\frac{dy}{dx} = \frac{-3b \cos^2 t \sin t}{3a \sin^2 t \cos t} = -\frac{b}{a} \cot t$ 1

$$\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = -\frac{b}{a}$$ 1

19. $\int (x+1)\sqrt{x^2-x+1}dx$

$$= \int\left[\frac{1}{2}(2x-1)+\frac{3}{2}\right]\sqrt{x^2-x+1}\,dx$$ 1

$$= \int\frac{1}{2}(2x-1)\sqrt{x^2-x+1}\,dx+\frac{3}{2}\int\sqrt{x^2-x+1}\,dx$$ ½

$$= \int\frac{1}{2}(2x-1)\sqrt{x^2-x+1}dx+\frac{3}{2}\int\sqrt{\left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,dx$$ 1

$$= \frac{1}{2}\times\frac{2}{3}(x^2-x+1)^{3/2}+\frac{3}{2}\left[\frac{(2x-1)}{4}\sqrt{x^2-x+1}+\frac{3}{8}\log\left[\frac{2x-1}{2}+\sqrt{x^2-x+1}\right]+C\right.$$ 1½

OR, $\frac{1}{3}(x^2-x+1)^{3/2}+\frac{3}{8}(2x-1)\sqrt{x^2-x+1}+\frac{9}{16}\log\left|\frac{(2x-1)}{2}+\sqrt{x^2-x+1}\right|+C$

20. Commutative: Let (a, b), (c, d) $\in$ A

(a, b) * (c, d) = (a + c, b + d) =(c + a, d + b) = (c, d) * (a, b)

* is commutative. 2

Associative: Let (a, b), (c, d), (e, f) $\in$ A

[(a, b) * (c, d)] * (e, f) = (a + c, b + d) * (e, f) = (a + c + e, b + d + f)

(a, b) * [(c, d) * (e, f)] = (a + b) * (c + e, d + f) = (a + c + e, b + d + f)

* is associative. 2

Identity element. Let $(e_1, e_2) \in$ A such that

$(a, b) * (e_1, e_2) = (e_1, e_2) * (a, b) = (a, b)$, for $(a, b) \in A$

$(a + e_1, b + e_2) = (e_1 + a, e_2 + b) = (a, b)$

$\Rightarrow a + e_1 = e_1 + a = a \Rightarrow e_1 = 0$

$\Rightarrow b + e_2 = e_2 + b = b \Rightarrow e_2 = 0$, but $0 \neq N$

$\therefore$ Identity element does not exist. 2

OR

one-one: Let $x_1, x_2 \in A$, $x_1 \neq 4 \neq x_2$

$f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 3}{x_1 - 4} = \frac{x_2 - 3}{x_2 - 4} \Rightarrow x_1 = x_2 \Rightarrow$ f is one – one 2½

Onto: Let $y \in B \quad y \neq -1$ with $y = f(x) \;\therefore\; y = \frac{x-3}{x-4}$

$\Rightarrow x = \frac{4y-3}{y-1}$, which is a real number $\because y \neq -1$.

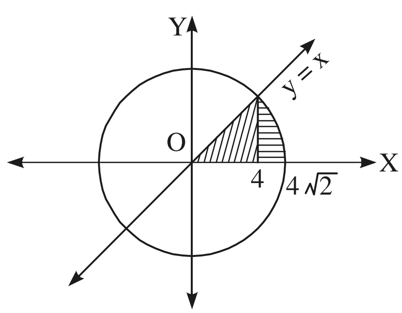
Also $x = 4 \Leftrightarrow \frac{4y-3}{y-1} = 4 \Leftrightarrow -3 = -4$, which is not possible

$\therefore x \neq 4, x \in A$ 2½

Thus for each $y \in B$, there exist $x \in A$ such that $y = f(x)$

$\therefore$ f is onto function with $f^{-1}(y) = \frac{4y-3}{y-1}$ 1

21.



Correct Figure 1

The line y = x intersects the circle at x = ±4 ½

Required Area $= \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2}\,dx$ 1½

$= \left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x\sqrt{32-x^2}}{2} + 16\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$ 1½

$= 8 + 0 + 16\sin^{-1}1 - 2(4) - 16\cdot\frac{\pi}{4}$

$= 8 + 8\pi - 8 - 4\pi = 4\pi$ sq. units 1½

22. $x\dfrac{dy}{dx} - y + x\sin\dfrac{y}{x} = 0$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{y}{x} - \sin\dfrac{y}{x}$ 1

$y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ 1

$v + x\dfrac{dv}{dx} = v - \sin v$

$\Rightarrow \int \operatorname{cosec} v \, dv = -\int \dfrac{1}{x} dx$ 1

$\Rightarrow \log|\operatorname{cosec} v - \cot v| = -\log|x| + \log C$

$\Rightarrow \log\left| x\left(\operatorname{cosec}\dfrac{y}{x} - \cot\dfrac{y}{x}\right)\right| = \log C \Rightarrow x\left(\operatorname{cosec}\dfrac{y}{x} - \cot\dfrac{y}{x}\right) = C$ 1

Put $y = \dfrac{\pi}{4}, x = 1$ we get

$C = \operatorname{cosec}\dfrac{\pi}{4} - \cot\dfrac{\pi}{4} = \sqrt{2} - 1$ 1

Solution: $x\left(\operatorname{cosec}\dfrac{y}{x} - \cot\dfrac{y}{x}\right) = \sqrt{2} - 1$ 1

OR

$x\dfrac{dy}{dx} + 2y = x^2 \Rightarrow \dfrac{dy}{dx} + \dfrac{2}{x}y = x$ 1

Integrating factor $= e^{\int \frac{2}{x} dx} = x^2$ 1

Solution: $yx^2 = \int x^3 \, dx + C$ 1

$\Rightarrow yx^2 = \dfrac{x^4}{4} + C$ 1 for y = 1, x = 1, we get $C = +\frac{3}{4}$ 1

Solution: $yx^2 = \frac{x^4}{4} + \frac{3}{4} \Rightarrow y = \frac{x^2}{4} + \frac{3}{4x^2}$ 1

23. Equation of line passing through origin and perpendicular to the given plane

$2x - 3y + 4z - 6 = 0$ is,

$\frac{x}{2} = \frac{y}{-3} = \frac{x}{4} = k$ (say) 1

$\therefore (x, y, z) = (2k, -3k, 4k)$ 1

It will lie on plane if

$2(2k) - 3(-3k) + 4(4k) = 6 \Rightarrow k = \frac{6}{29}$ 1

Coordinate of foot of perpendicular

$\left(\frac{12}{29}, -\frac{18}{29}, \frac{24}{29}\right)$ 1

length of perpendicular $= \sqrt{\frac{144}{841} + \frac{324}{841} + \frac{576}{841}}$ 1

$= \frac{6\sqrt{29}}{29}$ 1

24. E_1: Bolt produced by machine A.
E_2: Bolt produced by machine B.
E_3: Bolt produced by machine C. } 1

A: Bolt is defective.

$P(E_1) = \frac{25}{100}, P(E_2) = \frac{35}{100}, P(E_3) = \frac{40}{100}$ 1½

$P(A/E_1) = \frac{5}{100}, P(A/E_2) = \frac{4}{100}, P(A/E_3) = \frac{2}{100}$ 1½

$$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$$

$$= \frac{\frac{35}{100}\times\frac{4}{100}}{\frac{25}{100}\times\frac{5}{100}+\frac{35}{100}\times\frac{4}{100}+\frac{40}{100}\times\frac{2}{100}}$$ 1½

$$= \frac{28}{69}$$ ½

25. Let cost of shirt = Rs. x

Cost of print = Rs. y

The L.P.P. is

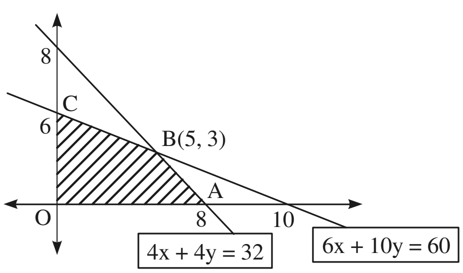
Minimise = z = 150 x + 200 y ½

Subject to

$6x + 10y \le 60$

$4x + 4y \le 32$ } 2

$x \ge 0,\ y \ge 0$



Correct graph 2

z at (B, 0) = Rs. 12,00

z at (5, 3) = Rs. 1350 } 1

z at (0, 6) = Rs. 1200

Cost is minimum when A works for 8 days or works for 6 days ½

26. Here

$x^2 + y^2 = k^2$ ½

Area (A) = $x \cdot y$ 1

$A = x \cdot \sqrt{k^2 - x^2}$

$$\frac{dA}{dx} = 1\cdot\sqrt{k^2 - x^2} + x\frac{(-2x)}{2\sqrt{k^2 - x^2}}$$ 1½

$$\frac{dA}{dx} = 0 \Rightarrow \sqrt{k^2 - x^2} = \frac{x^2}{\sqrt{k^2 - x^2}}$$

$$\Rightarrow k^2 - x^2 = x^2 \Rightarrow x = \frac{k}{\sqrt{2}}$$ 1

$$\left(\frac{d^2A}{dx^2}\right)_{x=\frac{k}{\sqrt{2}}} < 0$$ 1

Now $\left(\frac{k}{\sqrt{2}}\right)^2 + y^2 = k^2 \Rightarrow y = \frac{k}{\sqrt{2}}$ 1

Hence triangle is isoceles.