Design of Question Paper
Mathematics - Class X

Time: Three hours Max. Marks: 80

Weightage and distribution of marks over different dimensions of the question paper shall be as follows:

A. Weightage to content units

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Content Units</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Number systems</td>
<td>04</td>
</tr>
<tr>
<td>2.</td>
<td>Algebra</td>
<td>20</td>
</tr>
<tr>
<td>3.</td>
<td>Trigonometry</td>
<td>12</td>
</tr>
<tr>
<td>4.</td>
<td>Coordinate Geometry</td>
<td>08</td>
</tr>
<tr>
<td>5.</td>
<td>Geometry</td>
<td>16</td>
</tr>
<tr>
<td>6.</td>
<td>Mensuration</td>
<td>10</td>
</tr>
<tr>
<td>7.</td>
<td>Statistics &amp; Probability</td>
<td>10</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>80</td>
</tr>
</tbody>
</table>

B. Weightage to forms of questions

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Forms of Questions</th>
<th>Marks of each question</th>
<th>No. of Questions</th>
<th>Total marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Very Short answer questions (VSA)</td>
<td>01</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>2.</td>
<td>Short answer questions-I (SAI)</td>
<td>02</td>
<td>05</td>
<td>10</td>
</tr>
<tr>
<td>3.</td>
<td>Short answer questions-II (SAII)</td>
<td>03</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>4.</td>
<td>Long answer questions (LA)</td>
<td>06</td>
<td>05</td>
<td>30</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>30</td>
<td>80</td>
</tr>
</tbody>
</table>

C. Scheme of Options

All questions are compulsory. There is no overall choice in the question paper. However, internal choice has been provided in one question of two marks each, three questions of three marks each and two questions of six marks each.

D. Weightage to difficulty level of Questions

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Estimated difficulty level of questions</th>
<th>Percentage of marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Easy</td>
<td>15</td>
</tr>
<tr>
<td>2.</td>
<td>Average</td>
<td>70</td>
</tr>
<tr>
<td>3.</td>
<td>Difficult</td>
<td>15</td>
</tr>
</tbody>
</table>

Based on the above design, separate Sample papers along with their blue print and marking scheme have been included in this document for Board’s examination. The design of the question paper will remain the same whereas the blue print based on this design may change.
# Mathematics-X

## Blue Print I

<table>
<thead>
<tr>
<th>Unit</th>
<th>VSA (1 Mark) each</th>
<th>SAI (2 Marks) each</th>
<th>SA II (3 Marks) each</th>
<th>LA (6 Marks) each</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number systems</td>
<td>1(1)</td>
<td>—</td>
<td>3(1)</td>
<td>—</td>
<td>4(2)</td>
</tr>
<tr>
<td>Algebra</td>
<td>3(3)</td>
<td>2(1)</td>
<td>9(3)</td>
<td>6(1)</td>
<td>20(8)</td>
</tr>
<tr>
<td>Trigonometry</td>
<td>1(1)</td>
<td>2(1)</td>
<td>3(1)</td>
<td>6(1)</td>
<td>12(4)</td>
</tr>
<tr>
<td>Coordinate Geometry</td>
<td>—</td>
<td>2(1)</td>
<td>6(2)</td>
<td>—</td>
<td>8(3)</td>
</tr>
<tr>
<td>Geometry</td>
<td>2(2)</td>
<td>2(1)</td>
<td>6(2)</td>
<td>6(1)</td>
<td>16(6)</td>
</tr>
<tr>
<td>Mensuration</td>
<td>1(1)</td>
<td>—</td>
<td>3(1)</td>
<td>6(1)</td>
<td>10(3)</td>
</tr>
<tr>
<td>Statistic and Probability</td>
<td>2(2)</td>
<td>2(1)</td>
<td>—</td>
<td>6(1)</td>
<td>10(4)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>10(10)</strong></td>
<td><strong>10(5)</strong></td>
<td><strong>30(10)</strong></td>
<td><strong>30(5)</strong></td>
<td><strong>80(30)</strong></td>
</tr>
</tbody>
</table>
Sample Question Paper - I
Mathematics - Class X

Time : Three hours  Max.Marks :80

General Instructions.
1. All Questions are compulsory.
2. The question paper consists of thirty questions divided into 4 sections A, B, C and D. Section A comprises of ten questions of 01 mark each, section B comprises of five questions of 02 marks each, section C comprises of ten questions of 03 marks each and section D comprises of five questions of 06 marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in one question of 02 marks each, three questions of 03 marks each and two questions of 06 marks each. You have to attempt only one of the alternatives in all such questions.
5. In question on construction, drawings should be neat and exactly as per the given measurements.
6. Use of calculators is not permitted.However you may ask for mathematical tables.

Section A

1. Write the condition to be satisfied by $q$ so that a rational number $\frac{p}{q}$ has a terminating decimal expansion.
2. The sum and product of the zeroes of a quadratic polynomial are $-\frac{1}{2}$ and $-3$ repectively. What is the quadratic polynomial?
3. For what value of $k$ the quadratic equation $x^2 - kx + 4 = 0$ has equal roots?
4. Given that $\tan \theta = \frac{1}{\sqrt{5}}$, what is the value of $\frac{\csc^2 \theta - \sec^2 \theta}{\csc^2 \theta + \sec^2 \theta}$
5. Which term of the sequence 114, 109, 104 .... is the first negative term?
6. A cylinder, a cone and a hemisphere are of equal base and have the same height. What is the ratio in their volumes?

7. In the given figure, DE is parallel to BC and AD = 1cm, BD = 2cm. What is the ratio of the area of Δ ABC to the area of Δ ADE?

8. In the figure given below, PA and PB are tangents to the circle drawn from an external point P. CD is a third tangent touching the circle at Q. If PB = 10cm, and CQ = 2cm, what is the length of PC?

9. Cards each marked with one of the numbers 4,5,6,...,20 are placed in a box and mixed thoroughly. One card is drawn at random from the box. What is the probability of getting an even prime number?

10. A student draws a cumulative frequency curve for the marks obtained by 40 students of a class, as shown below. Find the median marks obtained by the students of the class.
Section B

11. Without drawing the graphs, state whether the following pair of linear equations will represent intersecting lines, coincident lines or parallel lines:

   \[6x - 3y + 10 = 0\]
   \[2x - y + 9 = 0\]

   Justify your answer.

12. Without using trigonometric tables, find the value of \(\frac{\cos 70^\circ}{\sin 20^\circ} + \cos 57^\circ \csc 33^\circ - 2 \cos 60^\circ\)

13. Find a point on the y-axis which is equidistant from the points A(6,5) and B (-4,3).

14. In the figure given below, AC is parallel to BD,

   \[\frac{AE}{CE} = \frac{DE}{BE}\] ? Justify your answer.

---

Section C

15. A bag contains 5 red, 8 green and 7 white balls. One ball is drawn at random from the bag, find the probability of getting

   (i) a white ball or a green ball.
   (ii) neither a green ball not a red ball.

OR

One card is drawn from a well shuffled deck of 52 playing cards. Find the probability of getting

   (i) a non-face card
   (ii) A black king or a red queen.

16. Using Euclid’s division algorithm, find the HCF of 56, 96 and 404.

OR

Prove that \(3 - \sqrt{5}\) is an irrational number

17. If two zeroes of the polynomial \(x^4+3x^3-20x^2-6x+36\) are \(\sqrt{2}\) and \(-\sqrt{2}\), find the other zeroes of the polynomial.

18. Draw the graph of the following pair of linear equations
$x + 3y = 6$
$2x - 3y = 12$

Hence find the area of the region bounded by the
$x = 0, \ y = 0$ and $2x - 3y = 12$

19. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs 200 for Ist day, Rs. 250 for second day, Rs. 300 for third day and so on. If the contractor pays Rs 27750 as penalty, find the number of days for which the construction work is delayed.

20. Prove that: \[\frac{1 + \cos A}{\sin A} + \frac{\sin A}{1 + \cos A} = 2 \csc A\]

OR

Prove that:

\[\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{\sin^2 A - \cos^2 A}\]

21. Observe the graph given below and state whether triangle ABC is scalene, isosceles or equilateral. Justify your answer. Also find its area.
22. Find the area of the quadrilateral whose vertices taken in order are A (-5, -3) B(-4, -6), C(2, -1) and D (1, 2).

23. Construct a \( \triangle ABC \) in which \( CA = 6 \text{cm}, AB = 5 \text{cm} \) and \( \angle BAC = 45^\circ \), then construct a triangle similar to the given triangle whose sides are \( \frac{6}{5} \) of the corresponding sides of the \( \triangle ABC \).

24. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre of the circle.

25. A square field and an equilateral triangular park have equal perimeters. If the cost of ploughing the field at rate of Rs 5/m\(^2\) is Rs 720, find the cost of maintaining the park at the rate of Rs 10/m\(^2\).

OR

An iron solid sphere of radius 3cm is melted and recast into small spherical balls of radius 1cm each. Assuming that there is no wastage in the process, find the number of small spherical balls made from the given sphere.

Section D

26. Some students arranged a picnic. The budget for food was Rs 240. Because four students of the group failed to go, the cost of food to each student got increased by Rs 5. How many students went for the picnic?

OR

A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500km away in time, it had to increase the speed by 250 km/h from the usual speed. Find its usual speed.

27. From the top of a building 100 m high, the angles of depression of the top and bottom of a tower are observed to be 45° and 60° respectively. Find the height of the tower. Also find the distance between the foot of the building and bottom of the tower.

OR

The angle of elevation of the top a tower at a point on the level ground is 30°. After walking a distance of 100m towards the foot of the tower along the horizontal line through the foot of the tower on the same level ground, the angle of elevation of the top of the tower is 60°. Find the height of the tower.

28. Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Using the above, solve the following:

A ladder reaches a window which is 12m above the ground on one side of the street. Keeping its foot at the same point, the ladder is turned to the other side of the street to reach a window 9m high. Find the width of the street if the length of the ladder is 15m.
29. The interior of building is in the form of a right circular cylinder of radius 7m and height 6m, surmounted by a right circular cone of same radius and of vertical angle 60°. Find the cost of painting the building from inside at the rate of Rs 30/m².

30. The following table shows the marks obtained by 100 students of class X in a school during a particular academic session. Find the mode of this distribution.

<table>
<thead>
<tr>
<th>Marks</th>
<th>No. of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 10</td>
<td>7</td>
</tr>
<tr>
<td>Less than 20</td>
<td>21</td>
</tr>
<tr>
<td>Less than 30</td>
<td>34</td>
</tr>
<tr>
<td>Less than 40</td>
<td>46</td>
</tr>
<tr>
<td>Less than 50</td>
<td>66</td>
</tr>
<tr>
<td>Less than 60</td>
<td>77</td>
</tr>
<tr>
<td>Less than 70</td>
<td>92</td>
</tr>
<tr>
<td>Less than 80</td>
<td>100</td>
</tr>
</tbody>
</table>
## Marking Scheme
### Sample Question Paper I
#### X-Mathmatics

<table>
<thead>
<tr>
<th>Q.No.</th>
<th>Value points</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Section A</strong></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>q should be expressible as $2^x \cdot 5^y$ where x, y are whole numbers</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>$2x^2 + x - 6$</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>$\pm 4$</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>$\frac{2}{3}$</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>$24^{\text{th}}$</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>$3 : 1 : 2$</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>$9 : 1$</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>$8 \text{ cm.}$</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>$0$</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>$55.$</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td><strong>Section B</strong></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Parallel lines</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>Here $\frac{a_1}{a_2} = 3$, $\frac{b_1}{b_2} = 3$, $\frac{c_1}{c_2} = \frac{10}{9}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>Given system of equations will represent parallel lines.</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>12</td>
<td>$\cos 70^\circ = \sin (90^\circ - 70^\circ) = \sin 20^\circ$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>$\cos 57^\circ = \sin (90^\circ - 57^\circ) = \sin 33^\circ$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>$\cos 60^\circ = \frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\cos 70^\circ + \cos 57^\circ \csc 33^\circ - 2 \cos 60^\circ$</td>
<td></td>
</tr>
</tbody>
</table>
\[
\frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\sin 33^\circ \csc 33^\circ}{2} - \frac{1}{2} = 1 + 1 - 1 = 1
\]

13. Let \((0, y)\) be a point on the y-axis, equidistant from \(A (6,5)\) and \(B (-4,3)\)

\[
PA = \sqrt{y^2 - 10y + 61}
\]
\[
P B = \sqrt{y^2 - 6y + 25}
\]

Now, \(PA = PB \Rightarrow (PA)^2 = (PB)^2\)

\[\begin{align*}
y^2 - 10y + 61 &= y^2 - 6y + 25 \\
y &= 9, \quad 1
\end{align*}\]

Required point is \((0, 9)\).

14. \(\triangle ACE \sim \triangle DBE\) (AA similarity)

\[
\frac{AC}{BD} = \frac{CE}{BE} = \frac{AE}{DE}
\]

15. (i) \(P\) (White or green ball) = \(\frac{15}{20} = \frac{3}{4}\)

(ii) \(P\) (Neither green nor red) = \(\frac{7}{20}\)

OR

(i) \(P\) (non-face card) = \(\frac{40}{52} = \frac{10}{13}\)

(ii) \(P\) (black king or red queen) = \(\frac{4}{52} = \frac{1}{13}\)
### Section C

<table>
<thead>
<tr>
<th>Q .No</th>
<th>Value Points</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 16    | Using Euclid’s division algorithm we have.  
  96 = 56x 1 + 40  
  56 = 40x 1 + 16  
  40 = 16x 2 + 8  
  16 = 8x 2 + 0  \[ \therefore \] HCF of 56 and 96 is 8.  | 2 |
|       | Now to find HCF of 56, 96 and 404 we apply Euclid’s division algorithm to  
  404 and 8 i.e.  
  404 = 8 x 50 + 4  
  8 = 4 x 2 + 0  \[ \therefore \] 4 is the required HCF.  | 1 |
| OR   | Let \(3 - \sqrt{5}\) be a rational number, say \(x\)  
  \[ \therefore \] \(3 - \sqrt{5} = x\)  
  \[ \Rightarrow \] \(\sqrt{5} = 3 - x\)  | ½ |
|      | Here R.H.S is a rational number, as both 3 and x are so  
  \[ \Rightarrow \] \(\sqrt{5}\) is a rational number  | ½ |
|      | proving that \(\sqrt{5}\) is not rational  | 1½ |
|      | Our supposition is wrong  | ½ |
|      | \[ \Rightarrow \] \(3 - \sqrt{5}\) is an irrational number  | ½ |
| 17.  | Since \(\sqrt{2}\) and \(-\sqrt{2}\) are two zeroes of the polynomial  
  \[ \therefore \] \((x - \sqrt{2})\) \((x + \sqrt{2})\) is a factor of the polynomial.  | 1 |
|      | By long division method  
  \(x^4 + 3x^3 - 20x^2 - 6x + 36 = (x^2 - 2) (x^2 + 3x - 18)\)  
  \[ = (x^2 - 2) (x + 6) (x - 3) \]  | 1 |
<p>|      | [ \therefore ] The other zeroes of the Polynomial are -6,3.  | 1 |</p>
<table>
<thead>
<tr>
<th>Q. No</th>
<th>Value Points</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.</td>
<td><img src="graph.png" alt="Graph" /> 1 Mark for drawing each of the two correct lines. Required Triangle is OAB, Area of triangle $\text{OAB} = \frac{1}{2} \times OA \times OB$ $= \frac{1}{2} \times 6 \times 4$ $= 12$ square Units</td>
<td>2</td>
</tr>
<tr>
<td>19.</td>
<td>Let the delay in construction work be for $n$ days Here $a = 200$, $d = 50$, $S_n = 27750$. $S_n = \frac{n}{2} [2a + (n-1) d]$ $27750 = \frac{n}{2} [2 \times 200 + (n-1) 50]$ $\Rightarrow n^2 + 7n - 1110 = 0$ $\Rightarrow (n + 37) (n - 30) = 0$ $n = -37$ (Rejected) or $n = 30$. $\therefore$ Delay in construction work was for 30 days</td>
<td>1.5</td>
</tr>
<tr>
<td>20.</td>
<td>$LHS = \frac{(1 + \cos A)^2 + (\sin A)^2}{\sin A (1 + \cos A)}$ $= \frac{2 + 2 \cos A}{\sin A (1 + \cos A)}$ $= \frac{2(1 + \cos A)}{\sin A (1 + \cos A)}$ $= \frac{2}{\sin A}$ $= 2 \csc A = RHS.$</td>
<td>2.5</td>
</tr>
</tbody>
</table>
Q. No | Value Points | Marks
--- | --- | ---
OR | \[
\text{LHS} = \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)}
\]
| 1 |
| | \[
= \frac{\sin^2 A + \cos^2 A + 2\sin A \cos A + \sin^2 A + \cos^2 A - 2\sin A \cos A}{\sin^2 A - \cos^2 A}
\]
| 1 |
| | \[
= \frac{2}{\sin^2 A - \cos^2 A} = \text{RHS}.
\]
| 1 |
21 | Scalene. | 1 |
| Justification: Coordinates of A, B and C are respectively (-3, -4), (3,0), (-5,0). | ½ |
| \[AB = \sqrt{52}\] | ½ |
| \[BC = \sqrt{8}\] | ½ |
| \[CA = \sqrt{20}\] | ½ |
| Clearly \(AB \neq BC \neq CA\). \(\therefore\) the given triangle is scalene. | ½ |
| Area = \(\frac{1}{2} \times \text{BC} \times \text{\(\perp\) from A on BC}\) | 1 |
| \[= \frac{1}{2} (8 \times 4) = 16 \text{ sq\text{•}u.}\] | 1 |
22. | Area of quad ABCD = area \(\triangle ABD\) + area \(\triangle BCD\). | ½ |
| area \(\triangle ABD\) = \(\frac{1}{2} \times [-5 (-6-2) - 4 (2+3) + (-3+6)]\). | 1 |
| \[= \frac{23}{2} \text{ sq\text{•}u.}\] | 1 |
| Area \(\triangle BCD\) = \(\frac{1}{2} \times [-4 (-1-2) + 2 (2 + 6) + 1 (-6+1)]\). | 1 |
23. For construction of \( \triangle ABC \)
   For construction of the required similar triangle  
   
   24.

   Since tangent is perpendicular to the radius:
   \( \angle SPO = \angle SRO = \angle OQT = 90° \)
   
   In right triangles OPS and ORS
   OS = OS (Common)
   OP = OR (radii of circle)

   \( \therefore \triangle OPS = \triangle ORS \) (RHS Congruence)

   \( \therefore \angle 1 = \angle 2 \)

   Similarly \( \angle 3 = \angle 4 \)

   Now \( \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180° \) 
   (Sum of angles on the same side of transversal)

   \( \Rightarrow \angle 2 + \angle 3 = 90° \)

   \( \therefore \angle SOT = 90° \)

25. Let the side of the square be ‘a’ meters
<table>
<thead>
<tr>
<th>Q. No</th>
<th>Value Points</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 x a² = 720 ⇒ a = 12m.</td>
<td>½</td>
<td></td>
</tr>
<tr>
<td>a = 12m. ½</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Perimeter of square = 48 m.</td>
<td>½</td>
<td></td>
</tr>
<tr>
<td>Perimeter of triangle = 48m.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Side of triangle = 16m.</td>
<td>½</td>
<td></td>
</tr>
<tr>
<td>Now Area of triangle = (\frac{\sqrt{3}}{4} \times 16 \times 16)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>= 64 (\sqrt{3}) m².</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Cost of maintaining the park</td>
<td></td>
<td></td>
</tr>
<tr>
<td>= Rs. (10 × 64 (\sqrt{3}))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>= Rs. (640 (\sqrt{3})).</td>
<td>½</td>
<td></td>
</tr>
<tr>
<td>OR</td>
<td></td>
<td></td>
</tr>
<tr>
<td>radius of sphere = 3cm.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume of sphere = (\frac{4}{3} \pi \times 3 \times 3 \times 3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>= 36 (\pi) cm³</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>radius of spherical ball = 1 cm.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume of one spherical ball = (\frac{4}{3} \pi \times 1 \times 1 \times 1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{4\pi}{3}) cm³</td>
<td>½</td>
<td></td>
</tr>
<tr>
<td>Let the number of small spherical balls be N.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\left(\frac{4\pi}{3}\right) \times N = 36 \pi)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>⇒ N = 27</td>
<td>½</td>
<td></td>
</tr>
</tbody>
</table>

**Section D**

26. Let the number of students who arranged the picnic be x.

∴ Cost of food for one student = \(\frac{240}{x}\) | 1 |

New cost of food for one student = \(\frac{240}{x - 4}\) | ½ |
Q .No | Value Points | Marks
---|---|---
240 \( \frac{240}{x-4} - \frac{240}{x} = 5 \) | 1½
\( \Rightarrow x^2 - 4x - 192 = 0 \) | 1
\( \Rightarrow (x - 16)(x + 12) = 0 \) | 1
\( \Rightarrow x = 16 \text{ or } x = -12 \) (Rejected) | ½
No of students who actually went for the picnic = 16 - 4 = 12 | ½

OR

Let the usual speed of plane be \( x \) km/hour

Time taken = \( \left( \frac{1500}{x} \right) \) hrs. with usual speed | 1

Time taken after increasing speed = \( \left( \frac{1500}{x+250} \right) \) hrs | ½

\( \frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2} \) | 1½

\( \Rightarrow x^2 + 250x - 750000 = 0 \) | 1

\( \Rightarrow (x + 1000)(x - 750) = 0 \) | 1

\( \Rightarrow x = 750 \text{ or } -1000 \) (Rejected) | ½

\( \therefore \) usual speed of plane = 750km/h. | ½

---

27.

Correct Figure | 1

In right \( \triangle BAC, \) \( \frac{AB}{AC} = \tan 60^\circ \)

\( \frac{100}{AC} = \tan 60^\circ \)

\( \Rightarrow AC = \left( \frac{100}{\sqrt{3}} \right) \text{ m.} \) | 1½
In right $\triangle BED$, \[
\frac{BE}{DE} = \tan 45^\circ = 1
\]

\[BE = DE\]

\[\therefore BE = \left(\frac{100}{\sqrt{3}}\right) \text{ m.} \quad 1\frac{1}{2}\]

Height of tower (CD) = AE

\[AE = AB - BE\]

\[= \left(100 - \frac{100}{\sqrt{3}}\right) \text{ m.} \quad 1\]

\[= 42.27 \text{ m.} \quad \frac{1}{2}\]

Distance between the foot the building and the bottom of the tower (AC) = 57.73 m. \quad \frac{1}{2}\]

OR

In right $\triangle BAC$, \[
\frac{AB}{AC} = \tan 30^\circ
\]

\[AB = (100 + AD) \times \frac{1}{\sqrt{3}} \quad (i) \quad 1\frac{1}{2}\]

In right $\triangle BAD$,

\[\frac{AB}{AD} = \tan 60^\circ\]

\[AB = AD \times \sqrt{3} \quad (ii) \quad 1\frac{1}{2}\]

From (i) and (ii) we get
<table>
<thead>
<tr>
<th>Q .No</th>
<th>Value Points</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[\frac{100 + AD}{\sqrt{3}} = AD \times \sqrt{3}]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100 + AD = 3 AD</td>
<td></td>
</tr>
<tr>
<td></td>
<td>⇒ AD = 50 m</td>
<td>1\frac{1}{2}</td>
</tr>
<tr>
<td></td>
<td>From (ii) AB = 50 \sqrt{3} m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 50 \times 1.732m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>or, AB = 86.6 m.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>28. Fig, Given, To Prove, Construction</td>
<td>(\frac{1}{2} \times 4 = 2)</td>
</tr>
<tr>
<td></td>
<td>Proof</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2nd part of the question:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AE = 9m.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>CE = 12m.</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td></td>
<td>width of street = 21 m.</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td></td>
<td>29. Correct Figure.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Internal curved surface area of cylinder</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>= 2\pi rh.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= (2\pi \times 7 \times 6 \text{ m}^2)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= \left(2 \times \frac{22}{7} \times 7 \times 6\right) \text{ m}^2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 264 \text{ m}^2</td>
<td>1\frac{1}{2}</td>
</tr>
</tbody>
</table>
In right \( \triangle OAB \), \( \frac{AB}{OB} = \sin 30^\circ \)

\[
\frac{7}{OB} = \frac{1}{2}
\]

\[\therefore \text{Slant height of cone (OB)} = 14 \text{m.} \]

Internal curved surface area of cone

\[= \pi rl\]

\[= \frac{22}{7} \times 7 \times 14\]

\[= 308 \text{m}^2.\]

Total Area to be painted = \((264 + 308)\)

\[= 572 \text{ m}^2.\]

Cost of painting = Rs \((30 \times 572)\)

\[= \text{Rs 17160.}\]

The given data can be written as -

<table>
<thead>
<tr>
<th>Marks</th>
<th>No of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 10</td>
<td>7</td>
</tr>
<tr>
<td>10 - 20</td>
<td>14</td>
</tr>
<tr>
<td>20 - 30</td>
<td>13</td>
</tr>
<tr>
<td>30 - 40</td>
<td>12</td>
</tr>
<tr>
<td>40 - 50</td>
<td>20</td>
</tr>
<tr>
<td>80 - 90</td>
<td>11</td>
</tr>
<tr>
<td>60 - 70</td>
<td>15</td>
</tr>
<tr>
<td>70 - 80</td>
<td>8</td>
</tr>
</tbody>
</table>

Mode = \(\mu = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h\)

\[\text{Here Modal class is 40 - 50}\]

\[\therefore \text{Mode} = 40 + \frac{(20 - 12)}{(2 \times 20 - 12 - 11)} \times 10\]

\[= 40 + \frac{80}{17}\]

\[= 44.7\]
## Mathematics-X
### Blue Print II

<table>
<thead>
<tr>
<th>Unit</th>
<th>VSA (1 Mark)</th>
<th>SA - I (2 Marks)</th>
<th>SA - II (3 Marks)</th>
<th>LA (6 Marks)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number systems</td>
<td>1(1)</td>
<td>-</td>
<td>3(1)</td>
<td>-</td>
<td>4(2)</td>
</tr>
<tr>
<td>Algebra</td>
<td>3(3)</td>
<td>2(1)</td>
<td>9(3)</td>
<td>6(1)</td>
<td>20(8)</td>
</tr>
<tr>
<td>Trigonometry</td>
<td>1(1)</td>
<td>2(1)</td>
<td>3(1)</td>
<td>6(1)</td>
<td>12(4)</td>
</tr>
<tr>
<td>Coordinate Geometry</td>
<td>-</td>
<td>2(1)</td>
<td>6(2)</td>
<td>-</td>
<td>8(3)</td>
</tr>
<tr>
<td>Geometry</td>
<td>2(2)</td>
<td>2(1)</td>
<td>6(2)</td>
<td>6(1)</td>
<td>16(6)</td>
</tr>
<tr>
<td>Mensuration</td>
<td>1(1)</td>
<td>-</td>
<td>3(1)</td>
<td>6(1)</td>
<td>10(3)</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>2(2)</td>
<td>2(1)</td>
<td>-</td>
<td>6(1)</td>
<td>10(4)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>10(10)</td>
<td>10(5)</td>
<td>30(10)</td>
<td>30(5)</td>
<td>80(30)</td>
</tr>
</tbody>
</table>
Section A

1. State the Fundamental Theorem of Arithmetic.

2. The graph of $y = f(x)$ is given below. Find the number of zeroes of $f(x)$. 

![Graph of $y = f(x)$]
3. Give an example of polynomials \( f(x) \), \( g(x) \), \( q(x) \), and \( r(x) \) satisfying 
\[
f(x) = g(x) \cdot q(x) + r(x) \quad \text{where deg } r(x) = 0.
\]

4. What is the nature of roots of the quadratic equation 
\[4x^2 - 12x - 9 = 0?
\]

5. If the adjoining figure is a sector of a circle of radius 10.5 cm,

find the perimeter of the sector. (Take \( \pi = \frac{22}{7} \))

6. The length of tangent from a point A at a distance of 5 cm from the centre of the circle is 4 cm. What will be the radius of the circle?

7. Which measure of central tendency is given by the x-coordinate of the point of intersection of the 'more than' ogive and 'less than' ogive?

8. A bag contains 5 red and 4 black balls. A ball is drawn at random from the bag. What is the probability of getting a black ball?

9. What is the distance between two parallel tangents of a circle of the radius 4 cm?

10. The height of a tower is 10m. Calculate the height of its shadow when Sun's altitude is 45°.

Section B

11. From your pocket money, you save Rs.1 on day 1, Rs. 2 on day 2, Rs. 3 on day 3 and so on. How much money will you save in the month of March 2008?

12. Express \( \sin 67° + \cos 75° \) in terms of trigonometric ratios of angles between 0° and 45°

OR

If \( A, B, C \) are interior angles of a \( \triangle ABC \), then show that
\[
\cos \left( \frac{B + C}{2} \right) = \sin \frac{A}{2}
\]

13. In the figure given below, DE // BC. If AD = 2.4 cm, DB = 3.6 cm and AC = 5 cm Find AE.

\[ \text{Diagram} \]

14. Find the values of x for which the distance between the point P (2,-3) and Q (x,5) is 10 units.

15. All cards of ace, jack and queen are removed from a deck of playing cards. One card is drawn at random from the remaining cards. find the probability that the card drawn is
   
   a) a face card
   
   b) not a face card

**Section C**

16. Find the zeroes of the quadratic polynomial \( x^2 + 5x + 6 \) and verify the relationship between the zeroes and the coefficients.

17. Prove that \( 5 + \sqrt{2} \) is irrational.

18. For what value or ‘k’ will the following pair of linear equations have infinitely many solutions
   
   \[ kx + 3y = k-3 \]
   
   \[ 12x + ky = k \]

   OR

   Solve for x and y

   \[
   \frac{5}{x} + \frac{1}{y} = 2
   \]

   \[
   \frac{6}{x} - \frac{3}{y} = 1
   \]

   \( \{x \neq 0, y \neq 0 \} \)
19. Determine an A.P. whose 3\textsuperscript{rd} term is 16 and when 5th term is subtracted from 7th term, we get 12.

OR

Find the sum of all three digit numbers which leave the remainder 3 when divided by 5.

20. Prove that
\[ \sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2\csc A \]

21. Prove that the points A(-3,0), B(1,-3) and C(4,1) are the vertices of an isosceles right triangle.

OR

For what value of ‘K’ the points A (1,5), B (K,1) and C (4,11) are collinear?

22. In what ratio does the point P(2,-5) divide the line segment joining A(-3,5) and B(4,-9)?

23. Construct a triangle similar to given ABC in which AB = 4 cm, BC = 6 cm and \( \angle ABC = 60^\circ \), such that each side of the new triangle is \( \frac{3}{4} \) of given \( \triangle ABC \).

24. The incircle of \( \triangle ABC \) touches the sides BC, CA and AB at D, E, and F respectively. If \( AB = AC \), prove that BD=CD.

25. PQRS is a square land of side 28m. Two semicircular grass covered portions are to be made on two of its opposite sides as shown in the figure. How much area will be left uncovered? \( \text{Take } \pi = \frac{22}{7} \)
26. Solve the following system of linear equations graphically:

\[ \begin{align*}
3x + y - 12 &= 0 \\
x - 3y + 6 &= 0
\end{align*} \]

Shade the region bounded by these lines and the x-axis. Also find the ratio of areas of triangles formed by given lines with x-axis and the y-axis.

27. There are two poles, one each on either bank of a river, just opposite to each other. One pole is 60m high. From the top of this pole, the angles of depression of the top and the foot of the other pole are 30° and 60° respectively. Find the width of the river and the height of the other pole.

28. Prove that the ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Use the above theorem, in the following.

The areas of two similar triangles are 81 cm² and 144 cm². If the largest side of the smaller triangle is 27 cm, find the largest side of the larger triangle.

OR

Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Use the above theorem, in the following.

If \( ABC \) is an equilateral triangle with \( AD \perp BC \), then \( AD^2 = 3 \ DC^2 \).

29. An iron pillar has lower part in the form of a right circular cylinder and the upper part in the form of a right circular cone. The radius of the base of each of the cone and cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar if 1 cm³ of iron weighs 7.5 grams. (Take \( \pi = \frac{22}{7} \))

OR

A container (open at the top) made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find

(i) the cost of milk when it is completely filled with milk at the rate of Rs 15 per litre.

(ii) the cost of metal sheet used, if it costs Rs 5 per 100 cm²

( Take \( \pi = 3.14 \))
30. The median of the following data is 20.75. Find the missing frequencies x and y, if the total frequency is 100.

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 5</td>
<td>7</td>
</tr>
<tr>
<td>5 - 10</td>
<td>10</td>
</tr>
<tr>
<td>10 - 15</td>
<td>x</td>
</tr>
<tr>
<td>15 - 20</td>
<td>13</td>
</tr>
<tr>
<td>20 - 25</td>
<td>y</td>
</tr>
<tr>
<td>25 - 30</td>
<td>10</td>
</tr>
<tr>
<td>30 - 35</td>
<td>14</td>
</tr>
<tr>
<td>35 - 40</td>
<td>9</td>
</tr>
<tr>
<td>Q .No</td>
<td>Value Points</td>
</tr>
<tr>
<td>-------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1.</td>
<td>Every Composite number can be factorised as a product of prime numbers. This factorisation is unique, apart from the order in which the prime factors occur.</td>
</tr>
<tr>
<td>2.</td>
<td>Two</td>
</tr>
<tr>
<td>3.</td>
<td>One such example : $f(x) = x^2 + 1, \ g(x) = x + 1, \ q(x) = (x-1)$ and $r(x) = 2$</td>
</tr>
<tr>
<td>4.</td>
<td>Real and Unequal</td>
</tr>
<tr>
<td>5.</td>
<td>32cm.</td>
</tr>
<tr>
<td>6.</td>
<td>3cm</td>
</tr>
<tr>
<td>7.</td>
<td>Median.</td>
</tr>
<tr>
<td>8.</td>
<td>$\frac{4}{9}$</td>
</tr>
<tr>
<td>9.</td>
<td>8 cm</td>
</tr>
<tr>
<td>10.</td>
<td>10 m.</td>
</tr>
<tr>
<td>11.</td>
<td>Let money saved be Rs $x$ [ x = 1+2+3+\ldots+31 \text{ (''31 days in March'')} ] [ S_n = \frac{(a+l)}{2} ]</td>
</tr>
<tr>
<td></td>
<td>[ S_n = \frac{31}{2} \left[ 1 + 31 \right] ] [ = \frac{31}{2} \times 2 \times 16 ] [ = 496 ]</td>
</tr>
<tr>
<td></td>
<td>Money Saved = Rs 496</td>
</tr>
<tr>
<td>12.</td>
<td>$\sin 67^\circ = \sin (90^\circ - 23^\circ)$ [ \cos 75^\circ = \cos (90^\circ - 15^\circ) ] [ \therefore \sin 67^\circ + \cos 75^\circ = \sin (90^\circ - 23^\circ) + \cos (90^\circ - 15^\circ) ]</td>
</tr>
<tr>
<td>Q.No</td>
<td>Value Points</td>
</tr>
<tr>
<td>------</td>
<td>--------------</td>
</tr>
<tr>
<td>12</td>
<td>( \cos 23° + \sin 15° )</td>
</tr>
<tr>
<td></td>
<td>OR</td>
</tr>
<tr>
<td></td>
<td>( \therefore A + B + C = 180° )</td>
</tr>
<tr>
<td></td>
<td>( \Rightarrow B + C = 180° - A )</td>
</tr>
<tr>
<td></td>
<td>( \therefore \frac{B + C}{2} = 90° - \frac{A}{2} )</td>
</tr>
<tr>
<td></td>
<td>( \therefore \text{LHS} = \cos (90° - \frac{A}{2}) )</td>
</tr>
<tr>
<td></td>
<td>= ( \sin \frac{A}{2} )</td>
</tr>
<tr>
<td></td>
<td>= ( \text{R.H.S} )</td>
</tr>
<tr>
<td>13</td>
<td>In ( \triangle ABC ), ( DE \parallel BC ),</td>
</tr>
<tr>
<td></td>
<td>( \therefore \text{By B.P.T,} )</td>
</tr>
<tr>
<td></td>
<td>( \frac{AE}{EC} = \frac{AD}{DB} )</td>
</tr>
<tr>
<td></td>
<td>( \Rightarrow \frac{AE}{AC - AE} = \frac{2.4}{3.6} = \frac{2}{3} )</td>
</tr>
<tr>
<td></td>
<td>( = 3AE = 2(AC - AE) )</td>
</tr>
<tr>
<td></td>
<td>( = 5AE = 2AC )</td>
</tr>
<tr>
<td></td>
<td>( = 2 \times 5\text{cm} )</td>
</tr>
<tr>
<td></td>
<td>( = AE = 2\text{cm} )</td>
</tr>
<tr>
<td>14</td>
<td>Given ( PQ = 10 \text{ Units} )</td>
</tr>
<tr>
<td></td>
<td>( \therefore \text{By Distance Formula} )</td>
</tr>
<tr>
<td></td>
<td>( \sqrt{(x - 2)^2 + (5 + 3)^2} = 10 )</td>
</tr>
<tr>
<td></td>
<td>( \Rightarrow (x - 2)^2 + 64 = 100 )</td>
</tr>
<tr>
<td></td>
<td>( \Rightarrow (x - 2)^2 = 36 )</td>
</tr>
<tr>
<td></td>
<td>( \Rightarrow x - 2 = +6, -6 )</td>
</tr>
<tr>
<td></td>
<td>( \Rightarrow x = 8, -4 )</td>
</tr>
<tr>
<td>Q.No</td>
<td>Value Points</td>
</tr>
<tr>
<td>------</td>
<td>--------------</td>
</tr>
</tbody>
</table>
| 15.  | **Total Number of Cards** = 52  
     | Cards removed (all aces, jacks and queens)  
     | = 12  
     | Cards Left = 52 - 12  
     | = 40  
     | **P (Event)** = Total number of favourable outcomes  
     | Total number of possible outcomes  
     | = \[
     \frac{\text{Total number of favourable outcomes}}{\text{Total number of possible outcomes}}
     \]  
     | ½  
     | ∴ **P (getting a face Card)** = \[
     \frac{4}{40} = \frac{1}{10}
     \]  
     | ½  
     | **P (Not getting a face Card)** =  
     | = \[
     1 - \frac{1}{10}
     \]  
     | = \[
     \frac{9}{10}
     \]  
     | ½ |
| 16.  | \(x^2 + 5x + 6 = (x+2) (x+3)\)  
     | Value of \(x^2 + 5x + 6\) is zero  
     | When \(x+2 = 0\) or \(x+3 = 0\)  
     | i.e. \(x = -2\) or \(x = -3\)  
     | Sum of zeroes = (-2) +(-3)  
     | = - 5  
     | = - \[
     \binom{5}{1}
     \]  
     | = - \[
     \frac{\text{Co-efficient of } x}{\text{Coefficient of } x^2}
     \]  
     | 1  
     | Product of zeroes = (-2) x (-3)  
     | = 6  
     | 269
<table>
<thead>
<tr>
<th>Q.No</th>
<th>Value Points</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

17. Suppose \( 5 + \sqrt{2} \) is a rational number, say \( n \).

\[ \Rightarrow \sqrt{2} = n - 5 \]

As \( n \) is rational and we know that \( 5 \) is rational,

\[ \therefore n - 5 \text{ is a rational number.} \]

\[ \therefore \sqrt{2} \text{ is a rational number} \]

Prove that \( \sqrt{2} \) is not a rational number

\[ \therefore \text{Our supposition is wrong} \]

Hence \( 5 + \sqrt{2} \) is an irrational number

18. For infinitely many solutions

\[ \frac{k}{12} = \frac{3}{k} = \frac{k - 3}{k} \quad (k \neq 0) \]

\[ \Rightarrow \frac{k}{12} = \frac{3}{k} \]

\[ = k^2 = 36 \]

\[ = k = \pm 6 \]

\[ \Rightarrow 3 = k - 3 \quad (k \neq 0) \]

\[ \Rightarrow k = 6 \]

The required value of \( k \) is 6.

OR

Put \( \frac{1}{x} = u \)

\[ \frac{1}{y} = v \]
5u + v = 2  (i)  
6u - 3v = 1  (ii)  

Multiplying equation (i) by 3 and adding to (ii) we get  
15u + 3v = 6  
6u - 3v = 1  

Adding 21u = 7  

\[ u = \frac{1}{21} \times 7 = \frac{1}{3} \]  

\[ v = \frac{1}{3} \]  

From (i)  
\[ v = 2 - 5u \]  

\[ = 2 - 5 \left( \frac{1}{3} \right) \]  

\[ = \frac{6 - 5}{3} \]  

\[ v = \frac{1}{3} \]  

\[ x = 3 \]  

\[ y = 3 \]  

19. Let the A.P be  
a, a+d, a+2d, - - - -  
a is the first term, d is the common difference  

It is given that  
\[ a + 2d = 16 \]  (1)  
\[ (a+6d) - (a+4d) = 12 \]  (2)  

From (2),  
\[ a + 6d - \left( a - 4d \right) = 12 \]  
\[ 2d = 12 \]  
\[ d = 6 \]  

Put d = 6 in (1)  
\[ a = 16 - 2d = 16 - 2 \times 6 = 4 \]
<table>
<thead>
<tr>
<th>Q.No</th>
<th>Value Points</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>= 16 - 12</td>
<td>½</td>
</tr>
<tr>
<td></td>
<td>= 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Required A.P. is 4, 10, 16, 22 - - - -</td>
<td>½</td>
</tr>
</tbody>
</table>

**OR**

The three digit numbers which when divided by 5 leave the reminder 3 are
103, 108, 113, - - - - , 998

Let their number be \(n\), then

\[ t_n = a + (n-1)d \]

\[ 998 = 103 + (n-1)5 \]

\[ = 103 + 5n - 5 \]

\[ 5n = 998 - 98 \]

\[ n = \frac{900}{5} - \frac{180}{5} \]

\[ n = 180 \]

Now,

\[ S_n = \frac{n}{2} [a + l] \]

\[ S_{180} = \frac{180}{2} [103 + 998] \]

\[ = 90 \times 1101 \]

\[ = 99090 \text{ Ans.} \]

20. L.H.S.

\[ = \sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} \]

\[ = \frac{\sec A - 1 + \sec A + 1}{\sqrt{\sec^2A - 1}} \]

\[ = \frac{2\sec A}{\sqrt{\tan^2A}} \quad (\therefore \sec^2A - 1 = \tan^2A) \]

\[ = \frac{1}{2} \]

\[ = 1 \]

\[ = \frac{1}{2} \]
\[
\begin{array}{c|c|c}
\text{Q .No} & \text{Value Points} & \text{Marks} \\
\hline
 & \frac{2 \sec A}{\tan A} & \frac{1}{2} \\
 & 2 \csc A & \frac{1}{2} \\
 & \text{R.H.S.} & \\
21. & \text{By distance formula} & \\
\text{AB} & = \sqrt{(1+3)^2 + (-3-0)^2} & \\
& = \sqrt{4^2 + (-3)^2} & \\
& = \sqrt{16 + 9} & \\
& = \sqrt{25} & \\
& = 5 \text{ units} & \\
\text{BC} & = \sqrt{(4-1)^2 + (1+3)^2} & \\
& = \sqrt{3^2 + 4^2} & \\
& = \sqrt{25} & \\
& = 5 \text{ units} & \\
\text{AC} & = \sqrt{(4+3)^2 + (1-0)^2} & \\
& = \sqrt{7^2 + 1^2} & \\
& = \sqrt{49 + 1} = \sqrt{50} = 5\sqrt{2} \text{ units} & 1 \\
\hline
\end{array}
\]

Since \( AB = BC = 5 \)

\( \triangle ABC \) is isosceles \((1)\) \( \frac{1}{2} \)

Now, \( (AB)^2 + (BC)^2 \)

= \( 5^2 + 5^2 \)

= \( 25 + 25 \)

= \( 50 \)

= \( (AC)^2 \)

\( \therefore \) By converse of pythagoras theorem
<table>
<thead>
<tr>
<th>Q.No</th>
<th>Value Points</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \triangle ABC ) is a right triangle</td>
<td>(2) 1</td>
</tr>
<tr>
<td></td>
<td>From (1) and (2) ( \triangle ABC ) is an isosceles right triangle</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>OR</td>
<td>( \text{We have} ) ( A(x_1, y_1) = A(1,5) ) ( B(x_2, y_2) = B(K,1) ) ( C(x_3, y_3) = C(4,11) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Since the given points are collinear the area of the triangle formed by them must be 0.</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>[ \frac{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)}{2} ] = 0</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td></td>
<td>=&gt; ( 1 \cdot (1-11) + K \cdot (11-5) + 4 \cdot (5-1) = 0 )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td></td>
<td>=&gt; -10 + 6K + 4(4) = 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>=&gt; 6K + 6 = 0</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td></td>
<td>=&gt; 6K = -6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( K = -1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The required value of ( K = -1 )</td>
<td>( \frac{1}{2} )</td>
</tr>
</tbody>
</table>

22. Let the point \( P(2, -5) \) divide the line segment joining \( A(-3,5) \) and \( B(4,-9) \) in the ratio \( K : 1 \) | \( \frac{1}{2} \) |

\[ K : 1 \]
\[ A(-3,5) \quad P(2,-5) \quad B(4,-9) \]

By Section formula

\[ 2 = \frac{4k - 3}{k + 1} \] | 1 |

\[ \therefore 2(k+1) = 4k - 3 \] | \( \frac{1}{2} \) |
<table>
<thead>
<tr>
<th>Q. No</th>
<th>Value Points</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2k = - 5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>k = $\frac{5}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>The required ratio is 5:2</td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>

23. For constructing $\triangle ABC$

For constructing similar triangle to $\triangle ABC$ with given dimensions 2

24.

![Diagram of a circle and tangents](image)

Since the lengths of tangents drawn from an external point to a circle are equal $\frac{1}{2}$

$\therefore$ we have

- $AF = AE$ - (1)
- $BF = BD$ - (2)
- $CD = CE$ - (3) $\frac{1}{2}$

Adding 1, 2 and 3, we get

$AF + BF + CD = AE + BD + CE$ 1

$AB + CD = AC + BD$ $\frac{1}{2}$

But $AB = AC$ (given) $\frac{1}{2}$

$\therefore CD = BD$ $\frac{1}{2}$
### Q.25

Area left uncovered

\[
= \text{Area (Square PQRS)} - 2 \times \text{(Area of Semicircle PAQ)}
\]

\[
= [(28 \times 28) - 2 \times \frac{1}{2} \times \left(\frac{22}{7} \times 14^2\right)] \text{m}^2
\]

\[
= (784 - 616) \text{ m}^2
\]

\[
= 168 \text{ m}^2
\]

### Q.26

**Section D**

We have

\[
3x + y - 12 = 0
\]

\[
y = 12 - 3x
\]

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>6</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

and

\[
x - 3y + 6 = 0
\]

\[
y = \frac{6 + x}{3}
\]

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>6</th>
<th>-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Q. No</td>
<td>Value Points</td>
<td>Marks</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>--------------</td>
<td>-------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Since the lines intersect at (3, 3), there is a unique solution given by ( x = 3, y = 3 )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Correct shaded portion</td>
<td>( \frac{1}{2} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Area of triangle ABC formed by lines with x-axis</td>
<td>( \frac{1}{2} \times 10 \times 3 = 15 \text{ sq. units} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Area of triangle BDE formed by lines with y-axis</td>
<td>( \frac{1}{2} \times 10 \times 3 = 15 \text{ sq units} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>∴ Ratio of these areas = 1 : 1</td>
<td>( \frac{1}{2} )</td>
<td></td>
</tr>
</tbody>
</table>

27.

Since the lines intersect at (3, 3), there is a unique solution given by \( x = 3, y = 3 \) 
Correct shaded portion
Area of triangle ABC formed by lines with x-axis
\[ = \frac{1}{2} \times 10 \times 3 = 15 \text{ sq. units} \]
Area of triangle BDE formed by lines with y-axis
\[ = \frac{1}{2} \times 10 \times 3 = 15 \text{ sq units} \]
∴ Ratio of these areas = 1 : 1
Correct figure

277
<table>
<thead>
<tr>
<th>Q.No</th>
<th>Value Points</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Let AB be the first pole and CD be the other one.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CA is the width of the river.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Draw DE \perp AB.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Let CD = h metre = AE</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BE = (60-h) m</td>
<td></td>
</tr>
</tbody>
</table>
|      | \[
\frac{60}{CA} = \sqrt{3}
\]
|      | \[
CA = \frac{60}{\sqrt{3}}
\]
|      | \[
= 20\sqrt{3}
\]
|      | 
|      | \[
\text{width of river } = 20\sqrt{3}
\]
|      | or
|      | \[
= 34.6\text{m}
\]
|      | 
|      | Now, In rt. (\triangle BAC) \[
\frac{BA}{CA} = \tan60^\circ
\]
|      | \[
\frac{60-h}{20\sqrt{3}} = \frac{1}{\sqrt{3}}
\]
|      | \[
60-h = 20
\]
|      | \[
h = 40
\]
|      | \[
\text{Height of the other pole } = 40\text{m}.
\]
Let the largest side of the larger triangle be \( x \) cm, then

\[
\frac{x^2}{27^2} = \frac{144}{81} \quad \text{(Using the theorem)}
\]

\[
\therefore \quad x = 36 \text{cm}
\]

OR

Let \( AC = a \) units

then \( DC = \frac{a}{2} \) units

In rt \( \triangle ADC \), by the above theorem

\[
AD^2 + DC^2 = AC^2
\]

\[
AD^2 = a^2 - \left( \frac{a}{2} \right)^2 = a^2 - \frac{a^2}{4}
\]

\[
AD^2 = 3 \left( \frac{a}{2} \right)^2 = 3DC^2
\]

\[
\therefore \quad AD^2 = 3DC^2
\]
Radius of base of Cylinder (r) = 8cm
Radius of base of Cone (r) = 8cm
Height of Cylinder (h) = 240cm
Height of Cone (H) = 36cm

Total volume of the pillar
= Volume of cylinder + volume of Cone
= \pi r^2 h + \frac{1}{3} \pi r^2 H
= \pi r^2 (h + \frac{1}{3} H)
= \frac{22}{7} \times 8 \times 8 \left[ 240 + \frac{1}{3} \times 36 \right] \text{ cm}^3
= \left( \frac{22}{7} \times 8 \times 8 \times 252 \right) \text{ cm}^3
= 50688 \text{ cm}^3
Weight of the pillar

\[ \text{Weight of the pillar} = (50688 \times \frac{7.5}{1000}) \text{ kg} \]
\[ = 380.16 \text{ kg} \]

OR

The Container is a frustum of cone
\[ h = 16\text{cm}, r = 8\text{cm}, R = 20\text{cm} \]

Volume of the container

\[ V = \frac{1}{3} \pi h (R^2 + Rr + r^2) \]
\[ = \frac{1}{3} \times 3.14 \times 16 ((20)^2 + 20(8) + (8)^2) \text{ cm}^3 \]
\[ = \frac{1}{3} \times 3.14 \times 16 (400 + 160 + 64) \text{ cm}^3 \]
\[ = \frac{1}{3} \times 3.14 \times 16 \times 624 \text{ cm}^3 \]
\[ = 3.14 \times 3328 \text{ cm}^3 \]
\[ = 10449.92 \text{ cm}^3 \]
\[ = 10.45 \text{ litres} \]

Cost of milk

\[ \text{Cost of milk} = \text{Rs} (10.45 \times 15) \]
\[ = \text{Rs} 156.75 \]

Now, slant height of the frustum of cone

\[ L = \sqrt{h^2 + (R-r)^2} \]
\[ = \sqrt{(16)^2 + (20-8)^2} \]
\[ = \sqrt{256 + 144} \]
\[ = 20\text{cm} \]
**Q.No** | **Value Points** | **Marks**
---|---|---
| **Total surface area of the container** | 1 |
| \[ = (\pi l (R+r) + \pi r^2) \] | |
| \[ = (3.14 \times 20 (20 + 8) + 3.14 (8)^2 \text{ cm}^2 \] | |
| \[ = 3.14 \times [20 \times 28 + 64]\text{ cm}^2 \] | |
| \[ = 3.14 \times 624 \] | |
| \[ = 1959.36 \text{ cm}^2 \] | |
| **Cost of metal Used** | 1 |
| \[ = Rs 1959.36 \times \frac{5}{100} \] | |
| \[ = Rs 19.5936 \times 5 \] | |
| \[ = Rs 97.968 \] | |
| \[ = Rs 98 \text{ (Approx.)} \] | |

### 30.

**Cumulative Frequency table**

<table>
<thead>
<tr>
<th>Class interval</th>
<th>frequency</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 5</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>5 - 10</td>
<td>10</td>
<td>17</td>
</tr>
<tr>
<td>10 - 15</td>
<td>x</td>
<td>17 + x</td>
</tr>
<tr>
<td>15 - 20</td>
<td>13</td>
<td>30 + x</td>
</tr>
<tr>
<td>20 - 25</td>
<td>y</td>
<td>30 + x + y</td>
</tr>
<tr>
<td>25 - 30</td>
<td>10</td>
<td>40 + x + y</td>
</tr>
<tr>
<td>30 - 35</td>
<td>14</td>
<td>54 + x + y</td>
</tr>
<tr>
<td>35 - 40</td>
<td>9</td>
<td>63 + x + y</td>
</tr>
</tbody>
</table>

Given \( n(\text{total frequency}) = 100 \)

\[ \Rightarrow 100 = 63 + x + y \]
\[ \Rightarrow x + y = 37 \quad (1) \]

The median is 20.75 which lies in the class 20-25

So, median class is 20-25

\[ \frac{1}{2} \]
\[ \begin{array}{|c|c|c|}
\hline
Q .No & Value Points & Marks \\
\hline
\therefore & l = 20 & \\
& f = y & \\
& c.f = 30 + x & \\
& h = 5 & ½ \\
\hline
Using formula, \\
Median = \left( l + \frac{n}{2} - c.f \right) \times h & 1 \\
\hline
20.75 = 20 + 50 - (30 + x) \times 5 & \\
\Rightarrow \frac{3y}{4} = \frac{(20 - x)}{y} & \\
\Rightarrow 3y = 400 - 20x & \\
\Rightarrow 20x + 3y = 400 & (2) 1\frac{1}{2} \\
Solving 1 and 2, we get \\
x = 17 & \\
y = 20 & 1 \\
\hline
\end{array} \]
Blue Print III  
X - Mathematics

<table>
<thead>
<tr>
<th>Unit</th>
<th>Form of Questions</th>
<th>VSA (1 Mark) each</th>
<th>SA - I (2 Marks) each</th>
<th>SA - II (3 Marks) each</th>
<th>LA (6 Marks) each</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number systems</td>
<td></td>
<td>1(1)</td>
<td>-</td>
<td>3(1)</td>
<td>-</td>
<td>4(2)</td>
</tr>
<tr>
<td>Algebra</td>
<td></td>
<td>3(3)</td>
<td>2(1)</td>
<td>9(3)</td>
<td>6(1)</td>
<td>20(8)</td>
</tr>
<tr>
<td>Trigonometry</td>
<td></td>
<td>1(1)</td>
<td>2(1)</td>
<td>3(1)</td>
<td>6(1)</td>
<td>12(4)</td>
</tr>
<tr>
<td>Coordinate Geometry</td>
<td></td>
<td></td>
<td>2(1)</td>
<td>6(2)</td>
<td>-</td>
<td>8(3)</td>
</tr>
<tr>
<td>Geometry</td>
<td></td>
<td>2(2)</td>
<td>2(1)</td>
<td>6(2)</td>
<td>6(1)</td>
<td>16(6)</td>
</tr>
<tr>
<td>Mensuration</td>
<td></td>
<td>1(1)</td>
<td>-</td>
<td>3(1)</td>
<td>6(1)</td>
<td>10(3)</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td></td>
<td>2(2)</td>
<td>2(1)</td>
<td>-</td>
<td>6(1)</td>
<td>10(4)</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>10(10)</td>
<td>10(5)</td>
<td>30(10)</td>
<td>30(5)</td>
<td>80(30)</td>
</tr>
</tbody>
</table>
Sample Question Paper III
Mathematics - Class X

Time : Three hours
Max. Marks : 80

General Instructions :

1. All Questions are compulsory.

2. The question paper consists of thirty questions divided into 4 sections A, B, C and D. Section A comprises of ten questions of 01 mark each, section B comprises of five questions of 02 marks each, section C comprises of ten questions of 03 marks each and section D comprises of five questions of 06 marks each.

3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4. There is no overall choice. However, internal choice has been provided in one question of 02 marks each, three questions of 03 marks each and two questions of 06 marks each. You have to attempt only one of the alternatives in all such questions.

5. In question on construction, drawings should be neat and exactly as per the given measurements.

6. Use of calculators is not permitted. However, you may ask for mathematical tables.

SECTION-A

1. Write 98 as product of its prime factors.

2. In fig. 1 the graph of a polynomial \( p(x) \) is given. Find the zeroes of the polynomial.
3. For what value of \( k \), the following pair of linear equations has infinitely many solutions?
   
   \[
   \begin{align*}
   10x + 5y - (k-5) &= 0 \\
   20x + 10y - k &= 0
   \end{align*}
   \]

4. What is the maximum value of \( \frac{1}{\sec \theta} \)?

5. If \( \tan A = \frac{3}{4} \) and \( A+B = 90^\circ \), then what is the value of cotB?

6. What is the ratio of the areas of a circle and an equilateral triangle whose diameter and a side are respectively equal?

7. Two tangents TP and TQ are drawn from an external point T to a circle with centre O, as shown in fig. 2. If they are inclined to each other at an angle of \( 100^\circ \) then what is the value of \( \angle POQ \)?
8. In fig. 3 what are the angles of depression from the observing positions O₁ and O₂ of the object at A?

9. A die is thrown once. What is the probability of getting a prime number?

10. What is the value of the median of the data using the graph in fig. 4, of less than ogive and more than ogive?

11. If the 10th term of an A.P. is 47 and its first term is 2, find the sum of its first 15 terms.

12. Justify the statement: “Tossing a coin is a fair way of deciding which team should get the batting first at the beginning of a cricket game.”

13. Find the solution of the pair of equations:

\[
\frac{3}{x} + \frac{8}{y} = -1, \quad \frac{1}{x} - \frac{2}{y} = 2, \quad x, y \neq 0
\]
14. The coordinates of the vertices of \( \triangle ABC \) are A(4, 1), B (-3, 2) and C (0, \( k \)). Given that the area of \( \triangle ABC \) is 12 unit\(^2\), find the value of \( k \).

15. Write a quadratic polynomial, sum of whose zeroes is \( 2 \sqrt{3} \) and their product is 2.

OR

What are the quotient and the remainder, when \( 3x^4 + 5x^3 - 7x^2 + 2x + 2 \) is divided by \( x^2 + 3x + 1 \)?

SECTION-C

16. If a student had walked 1 km/hr faster, he would have taken 15 minutes less to walk 3 km. Find the rate at which he was walking.

17. Show that \( 3 + 5 \sqrt{2} \) is an irrational number.

18. Find the value of \( k \) so that the following quadratic equation has equal roots:
   \[ 2x^2 - (k - 2)x + 1 = 0 \]

19. Construct a circle whose radius is equal to 4 cm. Let P be a point whose distance from its centre is 6 cm. Construct two tangents to it from P.

20. Prove that
   \[ \frac{\sin \theta}{\cot \theta + \csc \theta} = 2 + \frac{\sin \theta}{\cot \theta - \csc \theta} \]

OR

Evaluate
   \[ \frac{\sec 29^\circ}{\csc 61^\circ} + 2 \cot 8^\circ \cot 17^\circ \cot 45^\circ \cot 73^\circ \cot 82^\circ - 3 \left( \sin^2 38^\circ + \sin^2 52^\circ \right) \]

21. In fig. 5, \( \frac{XP}{PY} = \frac{XQ}{QZ} = 3 \), if the area of \( \triangle XYZ \) is 32 cm\(^2\), then find the area of the quadrilateral \( \text{PYZQ} \).

Fig. 5
A circle touches the side BC of a \( \triangle ABC \) at a point P and touches AB and AC when produced at Q and R respectively. Show that

\[
AQ = \frac{1}{2} \text{ (Perimeter of } \triangle ABC)\]

22. Find the ratio in which the line segment joining the points A (3, -6) and B(5,3) is divided by x-axis. Also find the coordinates of the point of intersection.

23. Find a relation between \( x \) and \( y \) such that the point P(\( x,y \)) is equidistant from the points A(2, 5) and B(-3, 7)

24. If in fig. 6, \( \triangle ABC \) and \( \triangle AMP \) are right angled at B and M respectively. prove that \( CA \times MP = PA \times BC \)

25. In Fig. 7, OAPB is a sector of a circle of radius 3.5 cm with the centre at O and \( \angle AOB = 120^\circ \). Find the length of OAPBO.

OR

Find the area of the shaded region of fig. 8 if the diameter of the circle with centre O is 28 cm and AQ = \( \frac{1}{4} \) AB.
SECTION-D

[26] Prove that in a triangle, if the square of one side is equal to the sum of the squares of the other two sides the angle opposite to the first side is a right angle. Using the converse of above, determine the length of an attitude of an equilateral triangle of side 2 cm.

[27] Form a pair of linear equations in two variables using the following information and solve it graphically.
Five years ago, Sagar was twice as old as Tiru. Ten year later Sagar’s age will be ten years more than Tiru’s age. Find their present ages. What was the age of Sagar when Tiru was born?

[28] From the top and foot of a tower 40m high, the angle of elevation of the top of a light house is found to be 30° and 60° respectively. Find the height of the lighthouse. Also find the distance of the top of the lighthouse from the foot of the tower.

[29] A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 100cm and the diameter of the hemispherical ends is 28cm. find the cost of polishing the surface of the solid at the rate of 5 paise per sq.cm.

OR

An open container made up of a metal sheet is in the form of a frustum of a cone of height 8cm with radii of its lower and upper ends as 4 cm and 10 cm respectively. Find the cost of oil which can completely fill the container a the rate of Rs. 50 per litre. Also, find the cost of metal used, if it costs Rs. 5 per 100 cm² (Use \(\pi = 3.14\))

[30] The mean of the following frequency table is 53. But the frequencies \(f_1\) and \(f_2\) in the classes 20-40 and 60-80 are missing. Find the missing frequencies.

<table>
<thead>
<tr>
<th>Age (in years)</th>
<th>0 - 20</th>
<th>20 - 40</th>
<th>40 - 60</th>
<th>60 - 80</th>
<th>80 -10</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of people</td>
<td>15</td>
<td>(f_1)</td>
<td>21</td>
<td>(f_2)</td>
<td>17</td>
<td>100</td>
</tr>
</tbody>
</table>

OR

Find the median of the following frequency distribution:

<table>
<thead>
<tr>
<th>Marks</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-100</td>
<td>2</td>
</tr>
<tr>
<td>100-200</td>
<td>5</td>
</tr>
<tr>
<td>200-300</td>
<td>9</td>
</tr>
<tr>
<td>300-400</td>
<td>12</td>
</tr>
<tr>
<td>400-500</td>
<td>17</td>
</tr>
<tr>
<td>500-600</td>
<td>20</td>
</tr>
<tr>
<td>600-700</td>
<td>15</td>
</tr>
<tr>
<td>700-800</td>
<td>9</td>
</tr>
<tr>
<td>800-900</td>
<td>7</td>
</tr>
<tr>
<td>900-1000</td>
<td>4</td>
</tr>
</tbody>
</table>
### MARKING SCHEME III

#### X MATHEMATICS

**SECTION A**

<table>
<thead>
<tr>
<th>Q. No</th>
<th>Value Points</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$2 \times 7^2$</td>
<td>1</td>
</tr>
<tr>
<td>2.</td>
<td>-3 and -1</td>
<td>1</td>
</tr>
<tr>
<td>3.</td>
<td>$k = 10$</td>
<td>1</td>
</tr>
<tr>
<td>4.</td>
<td>one</td>
<td>1</td>
</tr>
<tr>
<td>5.</td>
<td>$\frac{3}{4}$</td>
<td>1</td>
</tr>
<tr>
<td>6.</td>
<td>$\pi : \sqrt{3}$</td>
<td>1</td>
</tr>
<tr>
<td>7.</td>
<td>$\angle POQ = 80^\circ$</td>
<td>1</td>
</tr>
<tr>
<td>8.</td>
<td>$30^\circ, 45^\circ$</td>
<td>1</td>
</tr>
<tr>
<td>9.</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
</tr>
<tr>
<td>10.</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

**SECTION B**

11. Let $a$ be the first term and $d$ be the common difference of the A.P.

As we known that $a_n = a + (n - 1)d \Rightarrow 47 = 2 + 9d \Rightarrow d = 5$

$$s_{15} = \frac{15}{2} [2 \times 2 + (15 - 1) \times 5] = 555$$

12. When we toss a coin, the outcomes head or tail are equally likely.

So that the result of an individual coin toss is completely unpredictable.

Hence both teams get equal chance to bat first so the given statement is justified.

13. $\frac{3}{x} + \frac{8}{y} = -1$, ..........................(i)
<table>
<thead>
<tr>
<th>Q.No</th>
<th>Value Points</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) + (ii) x 4 = (\frac{7}{x} = 7) ⇒ (x = 1)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>From (ii) we get (1 - \frac{2}{y} = 2) ⇒ (y = -2)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>ABC = (\frac{1}{2} [4 (2-k) + (-3) (k-1) + 0 (1-2)] = 12) units²</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>(\Rightarrow \pm 12 = \frac{1}{2} [8 - 4k - 3k + 3])</td>
<td>(\frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>(-7k = 13, -35)</td>
<td>(\frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>(k = -\frac{13}{7}, 5)</td>
<td>(\frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>Let the quadratic polynomial be (x^2 + bx + c) and its zeroes be (\alpha) and (\beta) then we have</td>
<td>(\frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>(\alpha + \beta = 2\sqrt{3} = -b)</td>
<td>(\frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>(\alpha \beta = 2 = c)</td>
<td>(\frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>(\Rightarrow b = -2\sqrt{3}) and (c = 2), So a quadratic polynomial which satisfies the given conditions is (x^2 - 2\sqrt{3}x + 2)</td>
<td>(\frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>OR</td>
<td></td>
<td></td>
</tr>
<tr>
<td>By long division method</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Quotient = (3x^2 - 4x + 2)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Remainder = 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SECTION C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Let the original speed of walking of the student be (x) km/h</td>
<td>(\frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>Increased speed = ((x + 1)) km/h</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{3}{x} - \frac{3}{x+1} = \frac{15}{60})</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
Q. No | Value Points | Marks
---|---|---
293 | $4 \times 3(x + 1 - x) = x^2 + x$
 | $x^2 + x - 12 = 0$ | ½
 | $(x + 4)(x - 3) = 0$ | 
 | $x = 3, \ x = -4$ (rejected) | ½
 | His original speed was 3 km/h | ½

17. Let us assume, to the contrary, that $3 + 5\sqrt{2}$ is a rational number, say $x$
 | $5\sqrt{2} = x - 3$ | ½
 | $\sqrt{2} = \frac{x - 3}{5}$ | 
Now $x, 3$ and $5$ are all rational numbers
 | $\frac{x - 3}{5}$ is also a rational number | 
 | $\sqrt{2}$ is a rational number | ½
Prove : $\sqrt{2}$ is not a rational number | 1½
 | Our assumption is wrong | 
Hence $3 + 5\sqrt{2}$ is not a rational number | ½

18. Condition for $ax^2 + bx + c = 0$, have equal roots is
 | $b^2 - 4ac = 0$ | 
 | $[-(k - 2)]^2 - 4(2)(1) = 0$ | ½
 | $k^2 - 4k - 4 = 0$ | ½
 | $k = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-4)}}{2}$ | ½
 | $k = \frac{4 \pm 4\sqrt{2}}{2}$ | ½
<table>
<thead>
<tr>
<th>Q. No</th>
<th>Value Points</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 20.   | \[
\frac{\sin \theta}{\cot \theta + \cosec \theta} = 2 + \frac{\sin \theta}{\cot \theta + \cosec \theta}\]
|       | is true if \[
\frac{\sin \theta}{\cot \theta + \cosec \theta} - \frac{\sin \theta}{\cot \theta - \cosec \theta} = 2\]
|       | LHS = \[
\frac{\sin \theta \cot \theta - \sin \theta \cosec \theta - \sin \theta \cot \theta - \sin \theta \cosec \theta}{(\cot \theta + \cosec \theta)(\cot \theta - \cosec \theta)}\]
|       | = \[
\frac{-2 \sin \theta \cosec \theta}{\cot^2 \theta - \cosec^2 \theta}\]
|       | = \[
-2 \left( \sin \theta \times \frac{1}{\sin \theta} \right) \]
|       | = \[
-1 \]
|       | = \[
2\]
|       | = RHS
|       | i.e. LHS = RHS Hence proved

OR

\[
\sec 29^\circ = \sec (90^\circ - 61^\circ) = \cosec 61^\circ, \quad \cot 17^\circ = \cot (90^\circ - 73^\circ) = \tan 73^\circ
\]

\[
\cot 8^\circ = \cot (90^\circ - 82) = \tan 82^\circ \quad \sin^2 38^\circ = \sin^2 (90^\circ - 52^\circ) = \cos^2 52^\circ
\]

\[
\cot 45^\circ = 1
\]

\[
\therefore \frac{\sec 29^\circ}{\cosec 61^\circ} + 2 \cot 8^\circ \cot 17^\circ \cot 82^\circ \cot 73^\circ - 3 (\sin^2 38 + \sin^2 52^\circ)
\]

\[
= \frac{\cosec 61^\circ}{\cosec 61^\circ} + 2 \tan 82^\circ \tan 73^\circ \cot 82^\circ \cot 73^\circ - 3 (\cos^2 52 + \sin^2 52^\circ)
\]
\[
\begin{array}{c|c|c}
\text{Q. No} & \text{Value Points} & \text{Marks} \\
\hline
\Rightarrow & 1 + 2 - 3 & \frac{1}{2} \\
& = 0 & \\
\hline
21 & \frac{XP}{XY} = \frac{XQ}{XZ} = \frac{3}{4}, \quad \angle X = \angle X & \frac{1}{2} \\
& \Delta XPQ \sim \Delta XYZ & \frac{1}{2} \\
& \frac{XP}{XY} = \frac{3}{4} & \frac{1}{2} \\
& \frac{\text{ar } \Delta XPQ}{\text{ar } \Delta XYZ} = \left(\frac{3}{4}\right)^2 = \frac{9}{16} & \frac{1}{2} \\
& \text{ar } \Delta XPQ = \frac{9}{16} \times 32 = 18 \text{ cm}^2 & \frac{1}{2} \\
& \text{ar of quad PYZQ} = (32 - 18) \text{ cm}^2 = 14 \text{ cm}^2 & \frac{1}{2} \\
& \text{OR} & \\
& BP = BQ \text{ and } CP = CR & \frac{1}{2} \\
& AQ = AR & \frac{1}{2} \\
& AQ + AR = AB + BQ + AC + CR & \frac{1}{2} \\
& AQ + AQ = AB + BP + AC + PC & \frac{1}{2} \\
& 2AQ = AB + AC + BC & \frac{1}{2} \\
& AQ = \frac{1}{2} [AB + AC + BC] & \frac{1}{2} \\
& AQ = \frac{1}{2} \text{ (perimeter of } \Delta ABC) & \frac{1}{2} \\
\end{array}
\]
22. Let the ratio be \(k : 1\) then the coordinates of the point which divides AB in the ratio \(k : 1\) are

\[
\left( \frac{5k + 3}{k + 1}, \frac{3k - 6}{k + 1} \right)
\]

This point lies on x-axis

\[
\frac{3k - 6}{k + 1} = 0
\]

\[\Rightarrow k = 2\]

hence the ratio is 2:1

Putting \(k = 2\) we get the point of intersection

\[
\left( \frac{13}{3}, 0 \right)
\]

23. Let \(P(x, y)\) be equidistant from the point \(A(2, 5)\) and \(B(-3, 7)\).

\[
AP = BP \text{ so } AP^2 = BP^2
\]

\[
(x - 2)^2 + (y - 5)^2 = (x + 3)^2 + (y - 7)^2
\]

\[
x^2 - 4x + 4 + y^2 - 10y + 25 = x^2 + 6x + 9 + y^2 - 14y + 49
\]

\[
x^2 - 4x + 4 + y^2 - 10y + 25 = x^2 + 6x + 9 + y^2 - 14y + 49
\]

\[
x^2 - 10x + 9 = 29
\]

or \(10x - 4y + 29 = 0\) is the required relation

24. \(\triangle AMP \sim \triangle ABC\)

\[
\frac{PA}{CA} = \frac{MP}{BC}
\]

\[\Rightarrow CA \times MP = PA \times BC\]
25. length of OAPBO = length of arc BPA + 2 (radius)  

\[
= \frac{240}{360} \times 2 \times \frac{22}{7} \times 3.5 + 2 \times 3.5 
\]

\[
= \frac{2}{3} \times 2 \times \frac{22}{7} \times \frac{7}{2} + 7 
\]

\[
= 14 \frac{2}{3} + 7 = 21\frac{2}{3} 
\]

Length of OAPBO = 21\frac{2}{3} cm

**OR**

Diameter AQ = \( \frac{1}{4} \times 28 = 7 \text{ cm} \)  

=> \( r_1 = \frac{7}{2} \text{ cm} \)

\[ \text{area of shaded region} = \pi \left[ \left( \frac{7}{2} \right)^2 + \left( \frac{21}{2} \right)^2 \right] \]

\[ = \pi \times \left( \frac{7}{2} \right)^2 \left[ 1 + 3^2 \right] \]

\[ = \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times [10] \]

\[ = \frac{77 \times 5}{2} = \frac{385}{2} = 192.5 \text{ cm}^2 \]
26. Given, to prove, constant, figure
Proof of theorem

\[ AD \perp BC \]

\[(2a)^2 = h^2 + a^2\]

\[ h^2 = 4a^2 - a^2 \]

\[ h = \sqrt{3}a \]

\[ 2a = 2 \Rightarrow a = 1 \text{ cm} \]

\[ \therefore h = \sqrt{3} \text{ cm} \]

27. Present age of Sagar be \( x \) yrs & that of Tiru be \( y \) years.

\[
\begin{align*}
  x - 5 &= 2 \ (y - 5) \\
  x - 2y + 5 &= 0 \\
  x + 10 &= (y + 10) + 10 \\
  x - y - 10 &= 0
\end{align*}
\]

Equations: 1+1

| \( x \) | 5 | 15 | 25 |
| \( y \) | 5 | 10 | 15 |

| \( x \) | 15 | 20 | 25 |
| \( y \) | 5 | 10 | 15 |

Group: 1+1
Since the lines intersect at (25, 15)
Sagar’s present age = 25 yrs, Tiru’s present age = 15 yrs.

From graph it is clear that Sagar was 10 year's old, when Tiru was born.

For correct figure

Let $AE = h$ metre and $BE = CD = x$ metre

$$\frac{x}{h} = \cot 30^\circ = \sqrt{3}$$

$$\Rightarrow x = h \sqrt{3} \Rightarrow BE = CD = h \sqrt{3} \text{ m}$$

$$\frac{h + 40}{x} = \tan 60^\circ = \sqrt{3}$$

$$h + 40 = \sqrt{3} \times h \left( \sqrt{3} \right)$$

$$h = 20 \text{ m}$$

Height of lighthouse is $20 + 40 = 60 \text{ m}$

$$\frac{AD}{AC} = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\Rightarrow AC = 60 \times \frac{\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow AC = 40 \sqrt{3} \text{ m}$$

Hence the distance of the top of lighthouse from the foot of the tower is $40 \sqrt{3} \text{ m}$
<table>
<thead>
<tr>
<th>Question</th>
<th>Value Points</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 29.      | Radius of hemisphere = 14 cm.  
|          | Length of cylindrical part = \( [100 - 2(14)] = 72 \) cm  
|          | \( \text{radius of cylindrical part} = \text{radius of hemispherical ends} = 14 \) cm  
|          | Total area to be polished  
|          | \( = 2 (\text{C.S.A. of hemispherical ends}) + \text{C.S.A. of cylinder} \)  
|          | \( = 2 (2 \pi r^2) + 2 \pi rh \)  
|          | \( = 2 \times \frac{22}{7} \times 14 (2 \times 14 + 72) = 8800 \) cm\(^2\)  
|          | Cost of polishing the surface = Rs. 8800 x 0.05  
|          | \( = \text{Rs. 440} \)  
|          | \( \frac{1}{2} \)  
| OR       | The container is a frustum of a cone height 8 cm and radius of the bases 10 cm and 4 cm respectively  
|          | \( h = 8 \) cm, \( r_1 = 10 \) cm, \( r_2 = 4 \) cm  
|          | Slant height \( l = \sqrt{8^2 + (10 - 4)^2} = \sqrt{8^2 + 6^2} \) = 10 cm  
|          | Volume of container  
|          | \( = \frac{1}{3} \pi h \left( r_1^2 + r_2^2 + r_1 r_2 \right) \)  
|          | \( = \frac{1}{3} \times 3.14 \times 8 (100 + 16 + 40) \) cm\(^3\)  
|          | \( = \frac{1}{3} \times 3.14 \times 8 \times (156) \)  
|          | \( = 1306.24 \) cm\(^3\) = 1.31 / Litres (approx)  
|          | Quantity of oil = 1.31 / Litres  
|          | Cost of oil = Rs. (1.31 x 50)  
|          | \( = \text{Rs. 65.50} \)  
|          | \( \frac{1}{2} \) |
30. Surface area of the container (excluding the upper end)

\[ \pi \times [L(r_1 + r_2) + r_2^2] \]

\[ = 3.14 \times [10(10 + 4) + 16] \]

\[ = 3.14 \times 156 \]

\[ = 489.84 \text{ cm}^2 \]

cost of metal = Rs. \( \left( 489.84 \times \frac{5}{100} \right) = \text{Rs} \ 24.49 \)

<table>
<thead>
<tr>
<th>Age</th>
<th>Number of people ( f_i )</th>
<th>Class mark(( x_i ))</th>
<th>( x_i f_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-20</td>
<td>15</td>
<td>10</td>
<td>150</td>
</tr>
<tr>
<td>20-40</td>
<td>( f_1 )</td>
<td>30</td>
<td>30( f_1 )</td>
</tr>
<tr>
<td>40-60</td>
<td>21</td>
<td>50</td>
<td>1050</td>
</tr>
<tr>
<td>60-80</td>
<td>( f_2 )</td>
<td>70</td>
<td>70( f_2 )</td>
</tr>
<tr>
<td>80-100</td>
<td>17</td>
<td>90</td>
<td>1530</td>
</tr>
</tbody>
</table>

\[ \sum f_i = 53 + f_1 + f_2 = 100 \]

\[ \sum x_i f_i = 2730 + 30 f_1 + 70 f_2 \]

\[ \Rightarrow f_1 + f_2 = 47 \quad \ldots \quad (i) \]

\[ \bar{x} = \frac{\sum x_i f_i}{\sum f_i} \]

\[ = 53 = \frac{2730 + 30 f_1 + 70 f_2}{100} \]

\[ \Rightarrow 3 f_1 + 7 f_2 = 257 \quad \ldots \quad (ii) \]

Multiplying (i) by 3 and subtracting it from (ii) we get

\[ f_2 = 29 \]

\[ \frac{1}{2} \]

\[ 1 \]

\[ 1 \]
Put \( f_2 = 29 \) in (i) we get \( f_1 = 18 \)

Hence \( f_1 = 18 \) and \( f_2 = 29 \)

**OR**

<table>
<thead>
<tr>
<th>Age</th>
<th>frequency</th>
<th>Cumulative frequency (C.F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 -100</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>100 - 200</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>200 - 300</td>
<td>9</td>
<td>16</td>
</tr>
<tr>
<td>300 - 400</td>
<td>12</td>
<td>28</td>
</tr>
<tr>
<td>400 - 500</td>
<td>17</td>
<td>45</td>
</tr>
<tr>
<td>500 - 600</td>
<td>20</td>
<td>65</td>
</tr>
<tr>
<td>600 - 700</td>
<td>15</td>
<td>80</td>
</tr>
<tr>
<td>700 - 800</td>
<td>9</td>
<td>89</td>
</tr>
<tr>
<td>800 - 900</td>
<td>7</td>
<td>96</td>
</tr>
<tr>
<td>900 - 1000</td>
<td>4</td>
<td>100</td>
</tr>
</tbody>
</table>

\[
N = \sum f_i = 100 \quad \therefore \frac{N}{2} = 50
\]

\[
\therefore \text{median class is 500 -600}
\]

\[
l = 500, f = 20, F = 45, h = 100
\]

Hence

\[
\text{Median} = l + \left(\frac{\frac{N}{2} - F}{f}\right) \times h
\]

\[
\text{Median} = 500 + \left(\frac{50 - 45}{20}\right) \times 100
\]

\[
\text{Median} = 525
\]