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Senior School Certificate Examination

July — 2015 (Comptt.)

Marking Scheme — Mathematics (Outside Delhi) 65/1, 65/2, 65/3

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.
2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.
3. Alternative methods are accepted. Proportional marks are to be awarded.
4. In question (s) on differential equations, constant of integration has to be written.
5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.
6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.
7. Separate Marking Scheme for all the three sets has been given.
8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65/1
EXPECTED ANSWER VALE POINTS
SECTION A

	Marks
1. $3\vec{a} + 2\vec{b} = 7\vec{i} - 5\vec{j} + 4\vec{k}$	½
∴ D.R's are 7, -5, 4	½
2. $(2\vec{i} + 3\vec{j} + 2\vec{k}) \cdot (2\vec{i} + 2\vec{j} + \vec{k}) = 12$	½
$p = \frac{12}{ \vec{b} } = \frac{12}{3} = 4$	½
3. $\vec{r} = (\vec{i} + 2\vec{j} + 3\vec{k}) + \lambda(\vec{i} + 2\vec{j} - 5\vec{k})$	1
4. For singular matrix	
$4 \sin^2 x - 3 = 0$	½
$\sin x = \pm \frac{\sqrt{3}}{2} \Rightarrow x = \frac{2\pi}{3}$	½
5. $\int e^{2y} dy = \int x^3 dx$	½
$\Rightarrow \frac{e^{2y}}{2} = \frac{x^4}{4} + c$	½
6. I.F. = $\int \frac{1}{\sqrt{x}} dx$	½
$= e^{2\sqrt{x}}$	½

SECTION B

7. Let investment in first type of bonds be Rs x.	
∴ Investment in 2nd type = Rs (35000 - x)	½

$$\begin{pmatrix} x \\ 35000-x \end{pmatrix} \begin{pmatrix} \frac{8}{100} \\ \frac{10}{100} \end{pmatrix} = (3200) \quad 1\frac{1}{2}$$

$$\Rightarrow \left. \begin{aligned} \frac{8}{100}x + (35000-x)\frac{10}{100} &= 3200 \\ \Rightarrow x &= \text{Rs } 15000 \end{aligned} \right\} 1$$

$$\left. \begin{aligned} \therefore \text{Investment in first} &= \text{Rs } 15000 \\ \text{and in 2nd} &= \text{Rs } 20000 \end{aligned} \right\} 1$$

8. Getting $A' = \begin{pmatrix} 2 & 7 & 1 \\ 4 & 3 & -2 \\ -6 & 5 & 4 \end{pmatrix}$ 1

$$\text{Let } P = \frac{1}{2}(A + A') = \frac{1}{2} \begin{pmatrix} 4 & 11 & -5 \\ 11 & 6 & 3 \\ -5 & 3 & 8 \end{pmatrix} = \begin{pmatrix} 2 & 11/2 & -5/2 \\ 11/2 & 3 & 3/2 \\ -5/2 & 3/2 & 4 \end{pmatrix} \quad 1$$

Since $P' = P \therefore P$ is a symmetric matrix

$$\text{Let } Q = \frac{1}{2}(A - A') = \frac{1}{2} \begin{pmatrix} 0 & -3 & -7 \\ 3 & 0 & 7 \\ 7 & -7 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -3/2 & -7/2 \\ 3/2 & 0 & 7/2 \\ 7/2 & -7/2 & 0 \end{pmatrix} \quad 1$$

Since $Q' = -Q \therefore Q$ is skew symmetric

Also

$$P + Q = \begin{pmatrix} 2 & 11/2 & -5/2 \\ 11/2 & 3 & 3/2 \\ -5/2 & 3/2 & 4 \end{pmatrix} + \begin{pmatrix} 0 & -3/2 & -7/2 \\ 3/2 & 0 & 7/2 \\ 7/2 & -7/2 & 0 \end{pmatrix} = A \quad 1$$

OR

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 5 & -14 \end{pmatrix} \quad 1$$

$$\text{LHS} = (\text{AB})^{-1} = -\frac{1}{11} \begin{pmatrix} -14 & -5 \\ -5 & -1 \end{pmatrix} \text{ or } \frac{1}{11} \begin{pmatrix} 14 & 5 \\ 5 & 1 \end{pmatrix} \quad 1$$

$$\text{RHS} = \text{B}^{-1}\text{A}^{-1} = \frac{1}{11} \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix} \frac{1}{11} \begin{pmatrix} -4 & -3 \\ -1 & 2 \end{pmatrix} = \frac{1}{11} \begin{pmatrix} 14 & 5 \\ 5 & 1 \end{pmatrix} \quad 1+1$$

$\therefore \text{LHS} = \text{RHS}$

$$9. \begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

$$\text{R}_1 \rightarrow \text{R}_1 + \text{R}_2 + \text{R}_3,$$

$$\Rightarrow \begin{vmatrix} 3a-x & 3a-x & 3a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0 \quad 1$$

$$\text{C}_2 \rightarrow \text{C}_2 - \text{C}_1, \text{C}_3 \rightarrow \text{C}_3 - \text{C}_1$$

$$\Rightarrow \begin{vmatrix} 3a-x & 0 & 0 \\ a-x & 2x & 0 \\ a-x & 0 & 2x \end{vmatrix} = 0 \quad 1+1$$

$$\Rightarrow 4x^2(3a-x) = 0$$

$$\Rightarrow x = 0, x = 3a \quad 1$$

$$10. \text{I} = \int_0^{\pi/4} \log(1 + \tan x) dx \quad \dots(\text{i})$$

$$= \int_0^{\pi/4} \log \left[1 + \tan \left(\frac{\pi}{4} - x \right) \right] dx = \int_0^{\pi/4} \log \left[1 + \frac{1 - \tan x}{1 + \tan x} \right] dx \quad 1 + \frac{1}{2}$$

$$= \int_0^{\pi/4} [\log 2 - \log(1 + \tan x)] dx \quad \dots(\text{ii}) \quad 1$$

adding (i) and (ii) to get

$$2I = \log 2 \int_0^{\pi/4} 1 \cdot dx = \frac{\pi}{4} \log 2 \quad 1$$

$$\Rightarrow I = \frac{\pi}{8} \log 2 \quad 1/2$$

11. Writing $I = \int \frac{x}{(x^2+1)(x-1)} dx = \int \left(\frac{A}{x-1} + \frac{Bx+C}{x^2+1} \right) dx \quad 1$

$$= \int \frac{1/2}{x-1} dx + \int \frac{-\frac{1}{2}x + \frac{1}{2}}{x^2+1} dx \quad 1/2$$

$$= \frac{1}{2} \log |x-1| - \frac{1}{4} \log(x^2+1) + \frac{1}{2} \tan^{-1} x + C \quad 1/2$$

OR

$$I = \int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$$

Putting $x = \sin \theta$, $\therefore dx = \cos \theta d\theta$ and $x = 0$ then $\theta = 0$ } 1

$x \Rightarrow \frac{1}{\sqrt{2}}$ then $\theta = \frac{\pi}{4}$ }

$$I = \int_0^{\pi/4} \theta \cdot \frac{\cos \theta}{\cos^3 \theta} d\theta = \int_0^{\pi/4} \theta \cdot \sec^2 \theta d\theta \quad 1$$

$$= [\theta \tan \theta - \log |\sec \theta|]_0^{\pi/4} \quad 1$$

$$= \frac{\pi}{4} - \frac{1}{2} \log 2 \quad 1$$

12. (i) P (all four spades) = ${}^4C_4 \left(\frac{13}{52} \right)^4 \left(\frac{39}{52} \right)^0 = \frac{1}{256} \quad 2$

$$(ii) P(\text{only 2 are spades}) = {}^4C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^2 = \frac{27}{128} \quad 2$$

OR

$$n = 4, p = \frac{1}{6}, q = \frac{5}{6}$$

No. of successes

x	0	1	2	3	4	½
P(x)	${}^4C_0 \left(\frac{5}{6}\right)^4$	${}^4C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^3$	${}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$	${}^4C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)$	${}^4C_4 \left(\frac{1}{6}\right)^4$	} 2½
	$= \frac{625}{1296}$	$= \frac{500}{1296}$	$= \frac{150}{1296}$	$= \frac{20}{1296}$	$= \frac{1}{1296}$	
xP(x)	0	$\frac{500}{1296}$	$\frac{300}{1296}$	$\frac{60}{1296}$	$\frac{4}{1296}$	

$$\text{Mean} = \sum xP(x) = \frac{864}{1296} = \frac{2}{3} \quad 1$$

$$13. \text{LHS} = \vec{a} \cdot \{(\vec{b} + \vec{c}) \times \vec{d}\} = \vec{a} \cdot \{\vec{b} \times \vec{d} + \vec{c} \times \vec{d}\} \quad 1+1$$

$$= \vec{a} \cdot (\vec{b} \times \vec{d}) + \vec{a} \cdot (\vec{c} \times \vec{d}) \quad 1$$

$$= [\vec{a}, \vec{b}, \vec{d}] + [\vec{a}, \vec{c}, \vec{d}] \quad 1$$

$$14. \text{Here } \vec{a}_1 = 2\hat{i} - 5\hat{j} + \hat{k}, \quad \vec{a}_2 = 7\hat{i} - 6\hat{k}$$

$$\vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}, \quad \vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$$

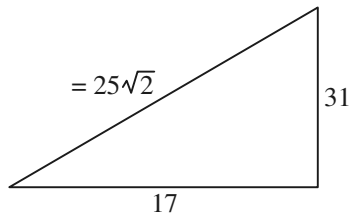
$$\vec{a}_2 - \vec{a}_1 = 5\hat{i} + 5\hat{j} - 7\hat{k} \quad 1$$

$$\vec{b}_1 \times \vec{b}_2 = -8\hat{i} + 4\hat{k} \quad 1$$

$$\text{SD} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \quad 1$$

$$= \frac{|-40-28|}{\sqrt{64+16}} = \frac{68}{\sqrt{80}} = \frac{17}{\sqrt{5}} \quad 1$$

15.



$$\text{LHS} = 2 \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1} \frac{2 \cdot \frac{1}{2}}{1 - \frac{1}{4}} + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1} \frac{4}{3} + \tan^{-1} \frac{1}{7} \quad 1\frac{1}{2}$$

$$= \tan^{-1} \frac{\frac{4}{3} + \frac{1}{7}}{1 - \frac{4}{3} \cdot \frac{1}{7}} = \tan^{-1} \frac{31}{17} \quad 1\frac{1}{2}$$

$$= \sin^{-1}\left(\frac{31}{25\sqrt{2}}\right) = \text{RHS} \quad 1$$

OR

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2} \tan^{-1} x$$

$$\Rightarrow \tan^{-1} 1 - \tan^{-1} x = \frac{1}{2} \tan^{-1} x \quad 1\frac{1}{2}$$

$$\Rightarrow \frac{3}{2} \tan^{-1} x = \frac{\pi}{4} \Rightarrow \tan^{-1} x = \frac{\pi}{6} \quad 1\frac{1}{2}$$

$$x = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}. \quad 1$$

$$16. \text{ LHL} = \lim_{x \rightarrow 0^-} f(x) = 2\lambda$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = 6$$

$$f(0) = 2\lambda$$

$$\Rightarrow 2\lambda = 6 \Rightarrow \lambda = 3 \quad 2$$

Differentiability

$$\text{LHD} = \lim_{h \rightarrow 0} \frac{f(0) - f(0-h)}{h} = \lim_{h \rightarrow 0} \frac{3(2) - 3((-h)^2 + 2)}{h} = \lim_{h \rightarrow 0} 3h = 0 \quad 1$$

$$\text{RHD} = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{(4h+6) - 3(2)}{h} = \lim_{h \rightarrow 0} 4 = 4 \quad 1/2$$

$$\text{LHD} \neq \text{RHD} \quad \therefore f(x) \text{ is not differentiable at } x = 0 \quad 1/2$$

17. $x = ae^t(\sin t + \cos t)$ and $y = ae^t(\sin t - \cos t)$

$$\frac{dx}{dt} = a[e^t(\cos t - \sin t) + e^t(\sin t + \cos t)] = -y + x \quad 1/2$$

$$\frac{dy}{dt} = a[e^t(\cos t + \sin t) + e^t(\sin t - \cos t)] = x + y \quad 1/2$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{x+y}{x-y} \quad 1$$

18. $y = Ae^{mx} + Be^{nx} \Rightarrow mAe^{mx} + nBe^{nx}$ 1

$$\frac{d^2y}{dx^2} = m^2Ae^{mx} + n^2Be^{nx} \quad 1$$

$$\begin{aligned} \text{LHS} &= \frac{d^2y}{dx^2} - (m+n) \frac{dy}{dx} + mny \\ &= m^2Ae^{mx} + n^2Be^{nx} - (m+n)\{mAe^{mx} + nBe^{nx}\} + mn\{Ae^{mx} + Be^{nx}\} \end{aligned} \quad 1$$

$$= Ae^{mx}(m^2 - m^2 - mn + mn) + Be^{nx}(n^2 - mn - n^2 + mn)$$

$$= 0 = \text{RHS.} \quad 1$$

$$\begin{aligned}
 19. \quad I &= \int \frac{x+3}{\sqrt{5-4x-2x^2}} dx = \int \frac{-\frac{1}{4}(-4-4x)+2}{\sqrt{5-4x-2x^2}} dx && 1 \\
 &= -\frac{1}{4} \cdot 2 \cdot \sqrt{5-4x-2x^2} + \frac{2}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(\frac{\sqrt{7}}{2}\right)^2 - (x+1)^2}} && 1+1 \\
 &= -\frac{1}{2} \sqrt{5-4x-2x^2} + \sqrt{2} \sin^{-1} \left(\frac{x+1}{\sqrt{7/2}} \right) + C && 1
 \end{aligned}$$

SECTION C

20. Here,

$$R = \left\{ \begin{array}{l} (1,1), (2,2), (3,3), (4,4), (5,5) \\ (1,3), (1,5), (2,4), (3,5) \\ (3,1), (5,1), (4,2), (5,3) \end{array} \right\} \quad 2$$

Clearly

(i) $\forall a \in A, (a, a) \in R \quad \therefore R$ is reflexive 1

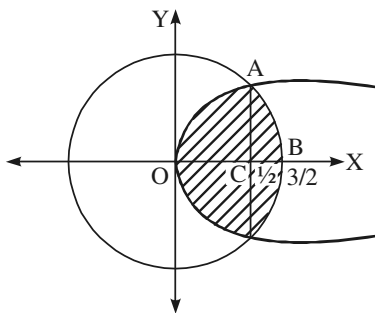
(ii) $\forall (a, b) \in A, (b, a) \in R \quad \therefore R$ is symmetric 1

(iii) $\forall (a, b), (b, c) \in R, (a, c) \in R \quad \therefore R$ is transitive 1

$\therefore R$ is an equivalence relation.

$$[1] = \{1, 3, 5\}, \quad [2] = \{2, 4\} \quad 1$$

21. $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$



Correct figure 1

Getting $x = \frac{1}{2}$ as point of intersection 1/2

$$A = 2 \left[2 \int_0^{1/2} \sqrt{x} dx + \int_{1/2}^{3/2} \sqrt{\frac{9}{4} - x^2} dx \right] \quad 1$$

$$= 2 \left[\left(\frac{4}{3} x^{3/2} \right)_0^{1/2} + \left(\frac{x}{2} \sqrt{9-x^2} + \frac{9}{8} \sin^{-1} \frac{2x}{3} \right)_{1/2}^{3/2} \right] \quad 1\frac{1}{2}$$

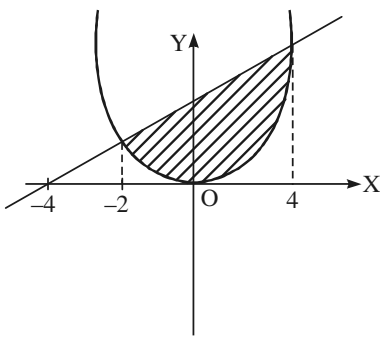
$$= 2 \left[\frac{2}{3\sqrt{2}} + \frac{9\pi}{16} - \frac{\sqrt{2}}{4} - \frac{9}{8} \sin^{-1} \frac{1}{3} \right] \quad 1$$

$$= \frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{3} \text{ sq. unit} \quad 1$$

OR

Correct figure 1

Getting $x = 4, -2$ as points of intersection $\frac{1}{2}$



$$A = \int_{-2}^4 \frac{1}{2} (3x + 12) dx - \int_{-2}^4 \frac{3}{4} x^2 dx \quad 1$$

$$= \frac{1}{2} \left(\frac{3x^2}{2} + 12x \right)_{-2}^4 - \frac{1}{4} (x^3)_{-2}^4 \quad 1\frac{1}{2}$$

$$= \frac{1}{2} (24 + 48 - 6 + 24) - \frac{1}{4} (64 + 8) \quad 1\frac{1}{2}$$

$$= 45 - 18 = 27 \text{ sq. units} \quad \frac{1}{2}$$

22. $\left(x \sin^2 \left(\frac{y}{x} \right) - y \right) dx + x dy = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{y - x \sin^2(y/x)}{x} = \frac{y}{x} - \sin^2 \left(\frac{y}{x} \right) \quad 1$$

$$v + x \frac{dv}{dx} = v - \sin^2 v \quad \text{where } \frac{y}{x} = v. \quad 1$$

$$\Rightarrow \int -\frac{dv}{\sin^2 v} = \int \frac{dx}{x} \quad \text{or} \quad \int -\operatorname{cosec}^2 v dv = \int \frac{dx}{x} \quad 1\frac{1}{2}$$

$$\cot v = \log x + C \text{ i.e., } \cot \frac{y}{x} = \log x + C \quad 1\frac{1}{2}$$

$$y = \frac{\pi}{4}, x = 1, \Rightarrow C = 1 \quad \frac{1}{2}$$

$$\Rightarrow \cot \frac{y}{x} = \log x + 1 \quad \frac{1}{2}$$

OR

$$\frac{dy}{dx} - 3 \cot x \cdot y = \sin 2x$$

$$\text{IF} = \int -3 \cot x \, dx = -3 \log \sin x = \text{cosec}^3 x \quad 1$$

\therefore Solution is

$$y \cdot \text{cosec}^3 x = \int \sin 2x \text{cosec}^3 x \, dx \quad 1\frac{1}{2}$$

$$= \int 2 \text{cosec} x \cot x \, dx \quad \frac{1}{2}$$

$$y \cdot \text{cosec}^3 x = -2 \text{cosec} x + C \quad 1\frac{1}{2}$$

$$\text{or } y = -2 \sin^2 x + C \sin^3 x$$

$$x = \frac{\pi}{2}, y = 2 \Rightarrow C = 4 \quad 1$$

$$\Rightarrow y = -2 \sin^2 x + 4 \sin^3 x \quad \frac{1}{2}$$

23. Equation of plane is

$$\{\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) - 7\} + \lambda \{\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) - 9\} = 0 \quad 1\frac{1}{2}$$

$$\Rightarrow \vec{r} \cdot \{(2 + 2\lambda)\hat{i} + (2 + 5\lambda)\hat{j} + (-3 + 3\lambda)\hat{k}\} = (7 + 9\lambda) \quad 1\frac{1}{2}$$

$$\text{x-intercept} = \text{y-intercept} \Rightarrow \frac{7 + 9\lambda}{2 + 2\lambda} = \frac{7 + 9\lambda}{-3 + 3\lambda} \quad 1$$

$$\Rightarrow \lambda = 5 \quad \frac{1}{2}$$

\therefore Eqn. of plane is

$$\vec{r} \cdot (12\hat{i} + 27\hat{j} + 12\hat{k}) = 52 \quad \frac{1}{2}$$

$$\text{and } 12x + 27y + 12z - 52 = 0 \quad 1$$

24. E_1 : student knows the answer

E_2 : student guesses the answer

A: answers correctly.

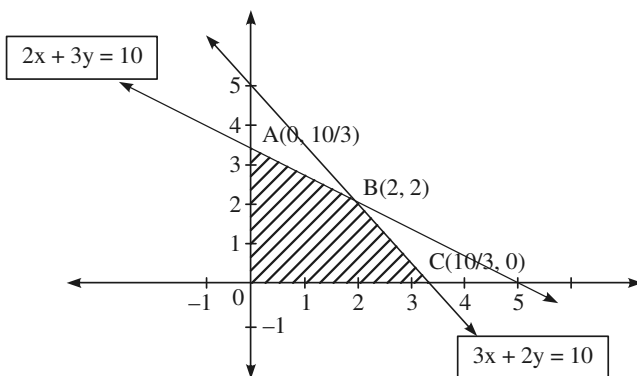
$$P(E_1) = \frac{3}{5}, \quad P(E_2) = \frac{2}{5} \quad 1$$

$$P\left(\frac{A}{E_1}\right) = 1, \quad P\left(\frac{A}{E_2}\right) = \frac{1}{3} \quad 1+1$$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)} \quad 1$$

$$= \frac{\frac{3}{5} \cdot 1}{\frac{3}{5} \cdot 1 + \frac{2}{5} \cdot \frac{1}{3}} = \frac{9}{11} \quad 1+1$$

25.



L.P.P. is Maximise $P = 24x + 18y$ $\frac{1}{2}$

$$\left. \begin{array}{l} \text{s.t. } 2x + 3y \leq 10 \\ 3x + 2y \leq 10 \\ x, y \geq 0 \end{array} \right\} \quad 2$$

Correct figure $\quad 2$

$$\left. \begin{array}{l} P(A) = \text{Rs } 60 \\ P(B) = \text{Rs } 84 \\ P(C) = \text{Rs } 80 \end{array} \right\} \quad \frac{1}{2}$$

\therefore Max. = 84 at (2, 2) $\quad 1$

$$26. \text{ Given: } s = 4\pi r^2 + 2 \left[\frac{x^2}{3} + 2x^2 + \frac{2x^2}{3} \right]$$

$$= 4\pi r^2 + 6x^2$$

1

$$V = \frac{4}{3}\pi r^3 + \frac{2x^3}{3}$$

$$V = \frac{4}{3}\pi r^3 + \frac{2}{3} \left(\frac{S - 4\pi r^2}{6} \right)^{3/2}$$

1

$$\frac{dv}{dr} = 4\pi r^2 + \left(\frac{S - 4\pi r^2}{6} \right)^{1/2} \left(\frac{-8\pi r}{6} \right)$$

1

$$\frac{dv}{dr} = 0 \Rightarrow r = \sqrt{\frac{S}{54 + 4\pi}}$$

1

$$\text{showing } \frac{d^2v}{dr^2} > 0$$

1

$$\therefore \text{ For } r = \sqrt{\frac{S}{54 + 4\pi}} \text{ volume is minimum}$$

$$\text{i.e., } (54 + 4\pi)r^2 = 4\pi r^2 + 6x^2$$

$$6x^2 = 54r^2 \Rightarrow x^2 = 9r^2 \Rightarrow x = 3r$$

1

QUESTION PAPER CODE 65/2
EXPECTED ANSWER VALE POINTS
SECTION A

	Marks
1. $\int e^{2y} dy = \int x^3 dx$	1/2
$\Rightarrow \frac{e^{2y}}{2} = \frac{x^4}{4} + c$	1/2
2. I.F. = $\int \frac{1}{\sqrt{x}} dx$	1/2
$= e^{2\sqrt{x}}$	1/2
3. $3\vec{a} + 2\vec{b} = 7\vec{i} - 5\vec{j} + 4\vec{k}$	1/2
\therefore D.R's are 7, -5, 4	1/2
4. $(2\vec{i} + 3\vec{j} + 2\vec{k}) \cdot (2\vec{i} + 2\vec{j} + \vec{k}) = 12$	1/2
$p = \frac{12}{ \vec{b} } = \frac{12}{3} = 4$	1/2
5. $\vec{r} = (\vec{i} + 2\vec{j} + 3\vec{k}) + \lambda(\vec{i} + 2\vec{j} - 5\vec{k})$	1
6. For singular matrix	
$4 \sin^2 x - 3 = 0$	1/2
$\sin x = \pm \frac{\sqrt{3}}{2} \Rightarrow x = \frac{2\pi}{3}$	1/2

SECTION B

7. $x = ae^t(\sin t + \cos t)$ and $y = ae^t(\sin t - \cos t)$	
$\frac{dx}{dt} = a[e^t(\cos t - \sin t) + e^t(\sin t + \cos t)] = -y + x$	1 1/2

$$\frac{dy}{dt} = a[e^t(\cos t + \sin t) + e^t(\sin t - \cos t)] = x + y \quad 1\frac{1}{2}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{x + y}{x - y} \quad 1$$

8. $y = Ae^{mx} + Be^{nx} \Rightarrow mAe^{mx} + nBe^{nx}$ 1

$$\frac{d^2y}{dx^2} = m^2Ae^{mx} + n^2Be^{nx} \quad 1$$

$$\begin{aligned} \text{LHS} &= \frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny \\ &= m^2Ae^{mx} + n^2Be^{nx} - (m+n)\{mAe^{mx} + nBe^{nx}\} + mn\{Ae^{mx} + Be^{nx}\} \\ &= Ae^{mx}(m^2 - m^2 - mn + mn) + Be^{nx}(n^2 - mn - n^2 + mn) \\ &= 0 = \text{RHS.} \end{aligned} \quad 1$$

9. $I = \int \frac{x+3}{\sqrt{5-4x-2x^2}} dx = \int \frac{-\frac{1}{4}(-4-4x)+2}{\sqrt{5-4x-2x^2}} dx$ 1

$$= -\frac{1}{4} \cdot 2 \cdot \sqrt{5-4x-2x^2} + \frac{2}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(\frac{\sqrt{7}}{2}\right)^2 - (x+1)^2}} \quad 1+1$$

$$= -\frac{1}{2}\sqrt{5-4x-2x^2} + \sqrt{2} \sin^{-1}\left(\frac{x+1}{\sqrt{7/2}}\right) + C \quad 1$$

10. Let investment in first type of bonds be Rs x.

\therefore Investment in 2nd type = Rs (35000 - x) 1/2

$$\left(\begin{matrix} x \\ 35000-x \end{matrix} \right) \left(\begin{matrix} \frac{8}{100} \\ \frac{10}{100} \end{matrix} \right) = (3200) \quad 1\frac{1}{2}$$

$$\Rightarrow \left. \begin{aligned} \frac{8}{100}x + (35000 - x)\frac{10}{100} &= 3200 \\ \Rightarrow x &= \text{Rs } 15000 \end{aligned} \right\} \quad 1$$

$$\therefore \left. \begin{aligned} \text{Investment in first} &= \text{Rs } 15000 \\ \text{and in 2nd} &= \text{Rs } 20000 \end{aligned} \right\} \quad 1$$

11. Getting $A' = \begin{pmatrix} 2 & 7 & 1 \\ 4 & 3 & -2 \\ -6 & 5 & 4 \end{pmatrix}$ 1

$$\text{Let } P = \frac{1}{2}(A + A') = \frac{1}{2} \begin{pmatrix} 4 & 11 & -5 \\ 11 & 6 & 3 \\ -5 & 3 & 8 \end{pmatrix} = \begin{pmatrix} 2 & 11/2 & -5/2 \\ 11/2 & 3 & 3/2 \\ -5/2 & 3/2 & 4 \end{pmatrix} \quad 1$$

Since $P' = P \therefore P$ is a symmetric matrix

$$\text{Let } Q = \frac{1}{2}(A - A') = \frac{1}{2} \begin{pmatrix} 0 & -3 & -7 \\ 3 & 0 & 7 \\ 7 & -7 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -3/2 & -7/2 \\ 3/2 & 0 & 7/2 \\ 7/2 & -7/2 & 0 \end{pmatrix} \quad 1$$

Since $Q' = -Q \therefore Q$ is skew symmetric

Also

$$P + Q = \begin{pmatrix} 2 & 11/2 & -5/2 \\ 11/2 & 3 & 3/2 \\ -5/2 & 3/2 & 4 \end{pmatrix} + \begin{pmatrix} 0 & -3/2 & -7/2 \\ 3/2 & 0 & 7/2 \\ 7/2 & -7/2 & 0 \end{pmatrix} = A \quad 1$$

OR

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 5 & -14 \end{pmatrix} \quad 1$$

$$\text{LHS} = (AB)^{-1} = -\frac{1}{11} \begin{pmatrix} -14 & -5 \\ -5 & -1 \end{pmatrix} \text{ or } \frac{1}{11} \begin{pmatrix} 14 & 5 \\ 5 & 1 \end{pmatrix} \quad 1$$

$$\text{RHS} = \mathbf{B}^{-1}\mathbf{A}^{-1} = \frac{1}{11} \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix} \frac{-1}{11} \begin{pmatrix} -4 & -3 \\ -1 & 2 \end{pmatrix} = \frac{1}{11} \begin{pmatrix} 14 & 5 \\ 5 & 1 \end{pmatrix} \quad 1+1$$

$\therefore \text{LHS} = \text{RHS}$

$$12. \begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 + R_2 + R_3,$$

$$\Rightarrow \begin{vmatrix} 3a-x & 3a-x & 3a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0 \quad 1$$

$$C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$$

$$\Rightarrow \begin{vmatrix} 3a-x & 0 & 0 \\ a-x & 2x & 0 \\ a-x & 0 & 2x \end{vmatrix} = 0 \quad 1+1$$

$$\Rightarrow 4x^2(3a-x) = 0$$

$$\Rightarrow x = 0, x = 3a \quad 1$$

$$13. I = \int_0^{\pi/4} \log(1 + \tan x) dx \quad \dots(i)$$

$$= \int_0^{\pi/4} \log \left[1 + \tan \left(\frac{\pi}{4} - x \right) \right] dx = \int_0^{\pi/4} \log \left[1 + \frac{1 - \tan x}{1 + \tan x} \right] dx \quad 1 + \frac{1}{2}$$

$$= \int_0^{\pi/4} [\log 2 - \log(1 + \tan x)] dx \quad \dots(ii) \quad 1$$

adding (i) and (ii) to get

$$2I = \log 2 \int_0^{\pi/4} 1 \cdot dx = \frac{\pi}{4} \log 2 \quad 1$$

$$\Rightarrow I = \frac{\pi}{8} \log 2 \quad \frac{1}{2}$$

$$14. \text{ Writing } I = \int \frac{x}{(x^2+1)(x-1)} dx = \int \left(\frac{A}{x-1} + \frac{Bx+C}{x^2+1} \right) dx \quad 1$$

$$= \int \frac{1/2}{x-1} dx + \int \frac{-\frac{1}{2}x + \frac{1}{2}}{x^2+1} dx \quad 1\frac{1}{2}$$

$$= \frac{1}{2} \log |x-1| - \frac{1}{4} \log(x^2+1) + \frac{1}{2} \tan^{-1} x + C \quad 1\frac{1}{2}$$

OR

$$I = \int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$$

$$\left. \begin{array}{l} \text{Putting } x = \sin \theta, \quad \therefore dx = \cos \theta d\theta \text{ and } x = 0 \text{ then } \theta = 0 \\ x \Rightarrow \frac{1}{\sqrt{2}} \text{ then } \theta = \frac{\pi}{4} \end{array} \right\} \quad 1$$

$$I = \int_0^{\pi/4} \theta \cdot \frac{\cos \theta}{\cos^3 \theta} d\theta = \int_0^{\pi/4} \theta \cdot \sec^2 \theta d\theta \quad 1$$

$$= [\theta \tan \theta - \log |\sec \theta|]_0^{\pi/4} \quad 1$$

$$= \frac{\pi}{4} - \frac{1}{2} \log 2 \quad 1$$

$$15. \text{ (i) } P(\text{all four spades}) = {}^4C_4 \left(\frac{13}{52} \right)^4 \left(\frac{39}{52} \right)^0 = \frac{1}{256} \quad 2$$

$$\text{(ii) } P(\text{only 2 are spades}) = {}^4C_2 \left(\frac{1}{4} \right)^2 \left(\frac{3}{4} \right)^2 = \frac{27}{128} \quad 2$$

OR

$$n = 4, p = \frac{1}{6}, q = \frac{5}{6}$$

No. of successes

x	0	1	2	3	4	$\frac{1}{2}$
P(x)	${}^4C_0 \left(\frac{5}{6}\right)^4$	${}^4C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^3$	${}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$	${}^4C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)$	${}^4C_4 \left(\frac{1}{6}\right)^4$	} $2\frac{1}{2}$
	$= \frac{625}{1296}$	$= \frac{500}{1296}$	$= \frac{150}{1296}$	$= \frac{20}{1296}$	$= \frac{1}{1296}$	
xP(x)	0	$\frac{500}{1296}$	$\frac{300}{1296}$	$\frac{60}{1296}$	$\frac{4}{1296}$	

$$\text{Mean} = \sum xP(x) = \frac{864}{1296} = \frac{2}{3}. \quad 1$$

$$16. \text{ LHS} = \vec{a} \cdot \{(\vec{b} + \vec{c}) \times \vec{d}\} = \vec{a} \cdot \{\vec{b} \times \vec{d} + \vec{c} \times \vec{d}\} \quad 1+1$$

$$= \vec{a} \cdot (\vec{b} \times \vec{d}) + \vec{a} \cdot (\vec{c} \times \vec{d}) \quad 1$$

$$= [\vec{a}, \vec{b}, \vec{d}] + [\vec{a}, \vec{c}, \vec{d}] \quad 1$$

$$17. \text{ Here } \vec{a}_1 = 2\hat{i} - 5\hat{j} + \hat{k}, \quad \vec{a}_2 = 7\hat{i} - 6\hat{k}$$

$$\vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}, \quad \vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$$

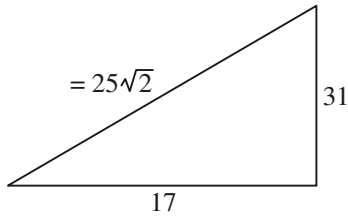
$$\vec{a}_2 - \vec{a}_1 = 5\hat{i} + 5\hat{j} - 7\hat{k} \quad 1$$

$$\vec{b}_1 \times \vec{b}_2 = -8\hat{i} + 4\hat{k} \quad 1$$

$$\text{SD} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \quad 1$$

$$= \frac{|-40 - 28|}{\sqrt{64 + 16}} = \frac{68}{\sqrt{80}} = \frac{17}{\sqrt{5}} \quad 1$$

18.



$$\text{LHS} = 2 \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1} \frac{2 \cdot \frac{1}{2}}{1 - \frac{1}{4}} + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1} \frac{4}{3} + \tan^{-1} \frac{1}{7} \quad 1\frac{1}{2}$$

$$= \tan^{-1} \frac{\frac{4}{3} + \frac{1}{7}}{1 - \frac{4}{3} \cdot \frac{1}{7}} = \tan^{-1} \frac{31}{17} \quad 1\frac{1}{2}$$

$$= \sin^{-1}\left(\frac{31}{25\sqrt{2}}\right) = \text{RHS} \quad 1$$

OR

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2} \tan^{-1} x$$

$$\Rightarrow \tan^{-1} 1 - \tan^{-1} x = \frac{1}{2} \tan^{-1} x \quad 1\frac{1}{2}$$

$$\Rightarrow \frac{3}{2} \tan^{-1} x = \frac{\pi}{4} \Rightarrow \tan^{-1} x = \frac{\pi}{6} \quad 1\frac{1}{2}$$

$$x = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \quad 1$$

$$19. \text{LHL} = \lim_{x \rightarrow 0^-} f(x) = 2\lambda$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = 6$$

$$f(0) = 2\lambda$$

$$\Rightarrow 2\lambda = 6 \Rightarrow \lambda = 3 \quad 2$$

Differentiability

$$\text{LHD} = \lim_{h \rightarrow 0} \frac{f(0) - f(0-h)}{h} = \lim_{h \rightarrow 0} \frac{3(2) - 3((-h)^2 + 2)}{h} = \lim_{h \rightarrow 0} 3h = 0 \quad 1$$

$$\text{RHD} = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{(4h+6) - 3(2)}{h} = \lim_{h \rightarrow 0} 4 = 4 \quad \frac{1}{2}$$

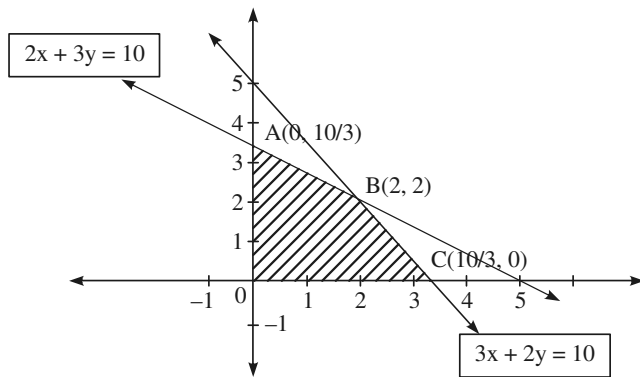
LHD \neq RHD $\therefore f(x)$ is not differentiable at $x = 0$ 1/2

SECTION C

20.

L.P.P. is Maximise $P = 24x + 18y$ 1/2

$$\left. \begin{aligned} \text{s.t. } 2x + 3y &\leq 10 \\ 3x + 2y &\leq 10 \\ x, y &\geq 0 \end{aligned} \right\} \quad 2$$



Correct figure 2

$$\left. \begin{aligned} P(A) &= \text{Rs } 60 \\ P(B) &= \text{Rs } 84 \\ P(C) &= \text{Rs } 80 \end{aligned} \right\} \quad \frac{1}{2}$$

\therefore Max. = 84 at (2, 2) 1

21. Given: $s = 4\pi r^2 + 2 \left[\frac{x^2}{3} + 2x^2 + \frac{2x^2}{3} \right]$

$$= 4\pi r^2 + 6x^2 \quad 1$$

$$V = \frac{4}{3}\pi r^3 + \frac{2x^3}{3}$$

$$V = \frac{4}{3}\pi r^3 + \frac{2}{3}\left(\frac{S - 4\pi r^2}{6}\right)^{3/2} \quad 1$$

$$\frac{dv}{dr} = 4\pi r^2 + \left(\frac{S - 4\pi r^2}{6}\right)^{1/2} \left(\frac{-8\pi r}{6}\right) \quad 1$$

$$\frac{dv}{dr} = 0 \Rightarrow r = \sqrt{\frac{S}{54 + 4\pi}} \quad 1$$

showing $\frac{d^2v}{dr^2} > 0$ 1

\therefore For $r = \sqrt{\frac{S}{54 + 4\pi}}$ volume is minimum

i.e., $(54 + 4\pi)r^2 = 4\pi r^2 + 6x^2$

$6x^2 = 54r^2 \Rightarrow x^2 = 9r^2 \Rightarrow x = 3r$ 1

22. Here,

$$R = \left\{ \begin{array}{l} (1,1), (2,2), (3,3), (4,4), (5,5) \\ (1,3), (1,5), (2,4), (3,5) \\ (3,1), (5,1), (4,2), (5,3) \end{array} \right\} \quad 2$$

Clearly

(i) $\forall a \in A, (a, a) \in R \therefore R$ is reflexive 1

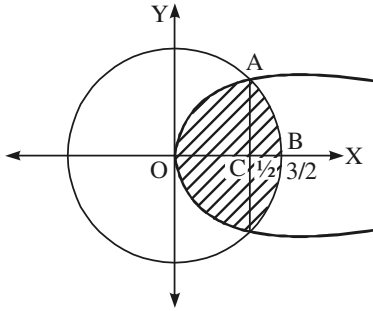
(ii) $\forall (a, b) \in A, (b, a) \in R \therefore R$ is symmetric 1

(iii) $\forall (a, b), (b, c) \in R, (a, c) \in R \therefore R$ is transitive 1

$\therefore R$ is an equivalence relation.

$[1] = \{1, 3, 5\}, \quad [2] = \{2, 4\}$ 1

23.



$$\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$$

Correct figure

1

Getting $x = \frac{1}{2}$ as point of intersection

1/2

$$A = 2 \left[2 \int_0^{1/2} \sqrt{x} dx + \int_{1/2}^{3/2} \sqrt{\frac{9}{4} - x^2} dx \right]$$

1

$$= 2 \left[\left(\frac{4}{3} x^{3/2} \right)_0^{1/2} + \left(\frac{x}{2} \sqrt{\frac{9}{4} - x^2} + \frac{9}{8} \sin^{-1} \frac{2x}{3} \right)_{1/2}^{3/2} \right]$$

1 1/2

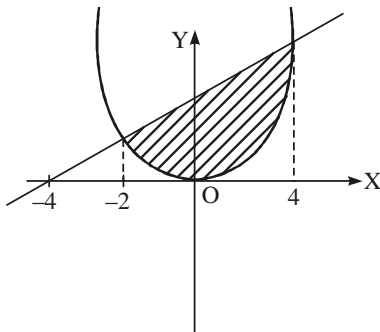
$$= 2 \left[\frac{2}{3\sqrt{2}} + \frac{9\pi}{16} - \frac{\sqrt{2}}{4} - \frac{9}{8} \sin^{-1} \frac{1}{3} \right]$$

1

$$= \frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{3} \text{ sq. unit}$$

1

OR



Correct figure

1

Getting $x = 4, -2$ as points of intersection

1/2

$$A = \int_{-2}^4 \frac{1}{2} (3x + 12) dx - \int_{-2}^4 \frac{3}{4} x^2 dx$$

1

$$= \frac{1}{2} \left(\frac{3x^2}{2} + 12x \right)_{-2}^4 - \frac{1}{4} (x^3)_{-2}^4$$

1 1/2

$$= \frac{1}{2} (24 + 48 - 6 + 24) - \frac{1}{4} (64 + 8)$$

1 1/2

$$= 45 - 18 = 27 \text{ sq. units}$$

1/2

$$24. \left(x \sin^2 \left(\frac{y}{x} \right) - y \right) dx + x dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - x \sin^2(y/x)}{x} = \frac{y}{x} - \sin^2 \left(\frac{y}{x} \right) \quad 1$$

$$v + x \frac{dv}{dx} = v - \sin^2 v \quad \text{where } \frac{y}{x} = v. \quad 1$$

$$\Rightarrow \int -\frac{dv}{\sin^2 v} = \int \frac{dx}{x} \quad \text{or} \quad \int -\operatorname{cosec}^2 v \, dv = \int \frac{dx}{x} \quad 1\frac{1}{2}$$

$$\cot v = \log x + C \quad \text{i.e.,} \quad \cot \frac{y}{x} = \log x + C \quad 1\frac{1}{2}$$

$$y = \frac{\pi}{4}, x = 1, \Rightarrow C = 1 \quad \frac{1}{2}$$

$$\Rightarrow \cot \frac{y}{x} = \log x + 1 \quad \frac{1}{2}$$

OR

$$\frac{dy}{dx} - 3 \cot x \cdot y = \sin 2x$$

$$\text{IF} = \int -3 \cot x \, dx = -3 \log \sin x = \operatorname{cosec}^3 x \quad 1$$

\(\therefore\) Solution is

$$y \cdot \operatorname{cosec}^3 x = \int \sin 2x \operatorname{cosec}^3 x \, dx \quad 1\frac{1}{2}$$

$$= \int 2 \operatorname{cosec} x \cot x \, dx \quad \frac{1}{2}$$

$$y \cdot \operatorname{cosec}^3 x = -2 \operatorname{cosec} x + C \quad 1\frac{1}{2}$$

$$\text{or } y = -2 \sin^2 x + C \sin^3 x$$

$$x = \frac{\pi}{2}, y = 2 \Rightarrow C = 4 \quad 1$$

$$\Rightarrow y = -2 \sin^2 x + 4 \sin^3 x \quad \frac{1}{2}$$

25. Equation of plane is

$$\{\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) - 7\} + \lambda \{\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) - 9\} = 0 \quad 1\frac{1}{2}$$

$$\Rightarrow \vec{r} \cdot \{(2 + 2\lambda)\hat{i} + (2 + 5\lambda)\hat{j} + (-3 + 3\lambda)\hat{k}\} = (7 + 9\lambda) \quad 1\frac{1}{2}$$

$$\text{x-intercept} = \text{y-intercept} \Rightarrow \frac{7 + 9\lambda}{2 + 2\lambda} = \frac{7 + 9\lambda}{-3 + 3\lambda} \quad 1$$

$$\Rightarrow \lambda = 5 \quad \frac{1}{2}$$

\therefore Eqn. of plane is

$$\vec{r} \cdot (12\hat{i} + 27\hat{j} + 12\hat{k}) = 52 \quad \frac{1}{2}$$

$$\text{and } 12x + 27y + 12z - 52 = 0 \quad 1$$

26. E_1 : student knows the answer

E_2 : student guesses the answer

A: answers correctly.

$$P(E_1) = \frac{3}{5}, \quad P(E_2) = \frac{2}{5} \quad 1$$

$$P\left(\frac{A}{E_1}\right) = 1, \quad P\left(\frac{A}{E_2}\right) = \frac{1}{3} \quad 1+1$$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)} \quad 1$$

$$= \frac{\frac{3}{5} \cdot 1}{\frac{3}{5} \cdot 1 + \frac{2}{5} \cdot \frac{1}{3}} = \frac{9}{11} \quad 1+1$$

QUESTION PAPER CODE 65/3
EXPECTED ANSWER/VALE POINTS
SECTION A

	Marks
1. $\vec{r} = (\vec{i} + 2\vec{j} + 3\vec{k}) + \lambda(\vec{i} + 2\vec{j} - 5\vec{k})$	1
2. For singular matrix	
$4 \sin^2 x - 3 = 0$	1/2
$\sin x = \pm \frac{\sqrt{3}}{2} \Rightarrow x = \frac{2\pi}{3}$	1/2
3. $\int e^{2y} dy = \int x^3 dx$	1/2
$\Rightarrow \frac{e^{2y}}{2} = \frac{x^4}{4} + c$	1/2
4. I.F. = $\int \frac{1}{\sqrt{x}} dx$	1/2
$= e^{2\sqrt{x}}$	1/2
5. $3\vec{a} + 2\vec{b} = 7\vec{i} - 5\vec{j} + 4\vec{k}$	1/2
\therefore D.R's are 7, -5, 4	1/2
6. $(2\vec{i} + 3\vec{j} + 2\vec{k}) \cdot (2\vec{i} + 2\vec{j} + \vec{k}) = 12$	1/2
$p = \frac{12}{ \vec{b} } = \frac{12}{3} = 4$	1/2

SECTION B

7. LHS = $\vec{a} \cdot \{(\vec{b} + \vec{c}) \times \vec{d}\} = \vec{a} \cdot \{\vec{b} \times \vec{d} + \vec{c} \times \vec{d}\}$	1+1
$= \vec{a} \cdot (\vec{b} \times \vec{d}) + \vec{a} \cdot (\vec{c} \times \vec{d})$	1
$= [\vec{a}, \vec{b}, \vec{d}] + [\vec{a}, \vec{c}, \vec{d}]$	1

8. Here $\vec{a}_1 = 2\hat{i} - 5\hat{j} + \hat{k}$, $\vec{a}_2 = 7\hat{i} - 6\hat{k}$

$$\vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}, \quad \vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$$

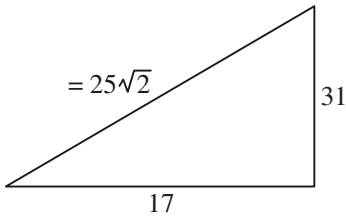
$$\vec{a}_2 - \vec{a}_1 = 5\hat{i} + 5\hat{j} - 7\hat{k} \quad 1$$

$$\vec{b}_1 \times \vec{b}_2 = -8\hat{i} + 4\hat{k} \quad 1$$

$$SD = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \quad 1$$

$$= \frac{|-40 - 28|}{\sqrt{64 + 16}} = \frac{68}{\sqrt{80}} = \frac{17}{\sqrt{5}} \quad 1$$

9.



$$LHS = 2 \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1} \frac{2 \cdot \frac{1}{2}}{1 - \frac{1}{4}} + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1} \frac{4}{3} + \tan^{-1} \frac{1}{7} \quad 1\frac{1}{2}$$

$$= \tan^{-1} \frac{\frac{4}{3} + \frac{1}{7}}{1 - \frac{4}{3} \cdot \frac{1}{7}} = \tan^{-1} \frac{31}{17} \quad 1\frac{1}{2}$$

$$= \sin^{-1}\left(\frac{31}{25\sqrt{2}}\right) = RHS \quad 1$$

OR

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2} \tan^{-1} x$$

$$\Rightarrow \tan^{-1} 1 - \tan^{-1} x = \frac{1}{2} \tan^{-1} x \quad 1\frac{1}{2}$$

$$\Rightarrow \frac{3}{2} \tan^{-1} x = \frac{\pi}{4} \Rightarrow \tan^{-1} x = \frac{\pi}{6} \quad 1\frac{1}{2}$$

$$x = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \quad 1$$

$$10. \text{LHL} = \lim_{x \rightarrow 0^-} f(x) = 2\lambda$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = 6$$

$$f(0) = 2\lambda$$

$$\Rightarrow 2\lambda = 6 \Rightarrow \lambda = 3$$

2

Differentiability

$$\text{LHD} = \lim_{h \rightarrow 0} \frac{f(0) - f(0-h)}{h} = \lim_{h \rightarrow 0} \frac{3(2) - 3((-h)^2 + 2)}{h} = \lim_{h \rightarrow 0} 3h = 0$$

1

$$\text{RHD} = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{(4h+6) - 3(2)}{h} = \lim_{h \rightarrow 0} 4 = 4$$

1/2

$$\text{LHD} \neq \text{RHD} \quad \therefore f(x) \text{ is not differentiable at } x = 0$$

1/2

$$11. x = ae^t(\sin t + \cos t) \text{ and } y = ae^t(\sin t - \cos t)$$

$$\frac{dx}{dt} = a[e^t(\cos t - \sin t) + e^t(\sin t + \cos t)] = -y + x$$

1 1/2

$$\frac{dy}{dt} = a[e^t(\cos t + \sin t) + e^t(\sin t - \cos t)] = x + y$$

1 1/2

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{x+y}{x-y}$$

1

$$12. y = Ae^{mx} + Be^{nx} \Rightarrow mAe^{mx} + nBe^{nx}$$

1

$$\frac{d^2y}{dx^2} = m^2Ae^{mx} + n^2Be^{nx}$$

1

$$\begin{aligned}
\text{LHS} &= \frac{d^2y}{dx^2} - (m+n) \frac{dy}{dx} + mny \\
&= m^2Ae^{mx} + n^2Be^{nx} - (m+n)\{mAe^{mx} + nBe^{nx}\} + mn\{Ae^{mx} + Be^{nx}\} \\
&= Ae^{mx}(m^2 - m^2 - mn + mn) + Be^{nx}(n^2 - mn - n^2 + mn) \\
&= 0 = \text{RHS.}
\end{aligned}$$

$$\begin{aligned}
13. \quad I &= \int \frac{x+3}{\sqrt{5-4x-2x^2}} dx = \int \frac{-\frac{1}{4}(-4-4x)+2}{\sqrt{5-4x-2x^2}} dx \\
&= -\frac{1}{4} \cdot 2 \cdot \sqrt{5-4x-2x^2} + \frac{2}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(\frac{\sqrt{7}}{2}\right)^2 - (x+1)^2}} \\
&= -\frac{1}{2} \sqrt{5-4x-2x^2} + \sqrt{2} \sin^{-1} \left(\frac{x+1}{\sqrt{7/2}} \right) + C
\end{aligned}$$

14. Let investment in first type of bonds be Rs x.

$$\therefore \text{Investment in 2nd type} = \text{Rs } (35000 - x)$$

$$\left(\begin{array}{c} x \\ 35000 - x \end{array} \right) \left(\begin{array}{c} \frac{8}{100} \\ \frac{10}{100} \end{array} \right) = (3200)$$

$$\begin{aligned}
&\Rightarrow \frac{8}{100}x + (35000 - x) \frac{10}{100} = 3200 \\
&\Rightarrow \qquad \qquad \qquad x = \text{Rs } 15000
\end{aligned}$$

$$\begin{aligned}
&\therefore \text{Investment in first} = \text{Rs } 15000 \\
&\text{and in 2nd} = \text{Rs } 20000
\end{aligned}$$

15. Getting $A' = \begin{pmatrix} 2 & 7 & 1 \\ 4 & 3 & -2 \\ -6 & 5 & 4 \end{pmatrix}$ 1

Let $P = \frac{1}{2}(A + A') = \frac{1}{2} \begin{pmatrix} 4 & 11 & -5 \\ 11 & 6 & 3 \\ -5 & 3 & 8 \end{pmatrix} = \begin{pmatrix} 2 & 11/2 & -5/2 \\ 11/2 & 3 & 3/2 \\ -5/2 & 3/2 & 4 \end{pmatrix}$ 1

Since $P' = P \therefore P$ is a symmetric matrix

Let $Q = \frac{1}{2}(A - A') = \frac{1}{2} \begin{pmatrix} 0 & -3 & -7 \\ 3 & 0 & 7 \\ 7 & -7 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -3/2 & -7/2 \\ 3/2 & 0 & 7/2 \\ 7/2 & -7/2 & 0 \end{pmatrix}$ 1

Since $Q' = -Q \therefore Q$ is skew symmetric

Also

$P + Q = \begin{pmatrix} 2 & 11/2 & -5/2 \\ 11/2 & 3 & 3/2 \\ -5/2 & 3/2 & 4 \end{pmatrix} + \begin{pmatrix} 0 & -3/2 & -7/2 \\ 3/2 & 0 & 7/2 \\ 7/2 & -7/2 & 0 \end{pmatrix} = A$ 1

OR

$AB = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 5 & -14 \end{pmatrix}$ 1

$LHS = (AB)^{-1} = -\frac{1}{11} \begin{pmatrix} -14 & -5 \\ -5 & -1 \end{pmatrix} \text{ or } \frac{1}{11} \begin{pmatrix} 14 & 5 \\ 5 & 1 \end{pmatrix}$ 1

$RHS = B^{-1}A^{-1} = \frac{1}{11} \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix} \frac{-1}{11} \begin{pmatrix} -4 & -3 \\ -1 & 2 \end{pmatrix} = \frac{1}{11} \begin{pmatrix} 14 & 5 \\ 5 & 1 \end{pmatrix}$ 1+1

$\therefore LHS = RHS$

16. $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

$$R_1 \rightarrow R_1 + R_2 + R_3,$$

$$\Rightarrow \begin{vmatrix} 3a-x & 3a-x & 3a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0 \quad 1$$

$$C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$$

$$\Rightarrow \begin{vmatrix} 3a-x & 0 & 0 \\ a-x & 2x & 0 \\ a-x & 0 & 2x \end{vmatrix} = 0 \quad 1+1$$

$$\Rightarrow 4x^2(3a-x) = 0$$

$$\Rightarrow x = 0, x = 3a \quad 1$$

$$17. I = \int_0^{\pi/4} \log(1 + \tan x) dx \quad \dots(i)$$

$$= \int_0^{\pi/4} \log \left[1 + \tan \left(\frac{\pi}{4} - x \right) \right] dx = \int_0^{\pi/4} \log \left[1 + \frac{1 - \tan x}{1 + \tan x} \right] dx \quad 1 + \frac{1}{2}$$

$$= \int_0^{\pi/4} [\log 2 - \log(1 + \tan x)] dx \quad \dots(ii) \quad 1$$

adding (i) and (ii) to get

$$2I = \log 2 \int_0^{\pi/4} 1 \cdot dx = \frac{\pi}{4} \log 2 \quad 1$$

$$\Rightarrow I = \frac{\pi}{8} \log 2 \quad \frac{1}{2}$$

$$18. \text{ Writing } I = \int \frac{x}{(x^2+1)(x-1)} dx = \int \left(\frac{A}{x-1} + \frac{Bx+C}{x^2+1} \right) dx \quad 1$$

$$= \int \frac{1/2}{x-1} dx + \int \frac{-\frac{1}{2}x + \frac{1}{2}}{x^2+1} dx \quad 1\frac{1}{2}$$

$$= \frac{1}{2} \log |x-1| - \frac{1}{4} \log(x^2+1) + \frac{1}{2} \tan^{-1} x + C \quad 1\frac{1}{2}$$

OR

$$I = \int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$$

$$\left. \begin{aligned} \text{Putting } x = \sin \theta, \quad \therefore dx = \cos \theta d\theta \text{ and } x = 0 \text{ then } \theta = 0 \\ x \Rightarrow \frac{1}{\sqrt{2}} \text{ then } \theta = \frac{\pi}{4} \end{aligned} \right\} \quad 1$$

$$I = \int_0^{\pi/4} \theta \cdot \frac{\cos \theta}{\cos^3 \theta} d\theta = \int_0^{\pi/4} \theta \cdot \sec^2 \theta d\theta \quad 1$$

$$= [\theta \tan \theta - \log |\sec \theta|]_0^{\pi/4} \quad 1$$

$$= \frac{\pi}{4} - \frac{1}{2} \log 2 \quad 1$$

$$19. \text{ (i) } P(\text{all four spades}) = {}^4C_4 \left(\frac{13}{52}\right)^4 \left(\frac{39}{52}\right)^0 = \frac{1}{256} \quad 2$$

$$\text{(ii) } P(\text{only 2 are spades}) = {}^4C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^2 = \frac{27}{128} \quad 2$$

OR

$$n = 4, p = \frac{1}{6}, q = \frac{5}{6}$$

No. of successes

x	0	1	2	3	4	½
P(x)	${}^4C_0 \left(\frac{5}{6}\right)^4$	${}^4C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^3$	${}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$	${}^4C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)$	${}^4C_4 \left(\frac{1}{6}\right)^4$	} 2½
	$= \frac{625}{1296}$	$= \frac{500}{1296}$	$= \frac{150}{1296}$	$= \frac{20}{1296}$	$= \frac{1}{1296}$	
xP(x)	0	$\frac{500}{1296}$	$\frac{300}{1296}$	$\frac{60}{1296}$	$\frac{4}{1296}$	
Mean = $\sum xP(x)$	$= \frac{864}{1296} = \frac{2}{3}$					1

SECTION C

20. Equation of plane is

$$\{\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) - 7\} + \lambda \{\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) - 9\} = 0 \quad 1\frac{1}{2}$$

$$\Rightarrow \vec{r} \cdot \{(2 + 2\lambda)\hat{i} + (2 + 5\lambda)\hat{j} + (-3 + 3\lambda)\hat{k}\} = (7 + 9\lambda) \quad 1\frac{1}{2}$$

$$\text{x-intercept} = \text{y-intercept} \Rightarrow \frac{7 + 9\lambda}{2 + 2\lambda} = \frac{7 + 9\lambda}{-3 + 3\lambda} \quad 1$$

$$\Rightarrow \lambda = 5 \quad \frac{1}{2}$$

∴ Eqn. of plane is

$$\vec{r} \cdot (12\hat{i} + 27\hat{j} + 12\hat{k}) = 52 \quad \frac{1}{2}$$

$$\text{and } 12x + 27y + 12z - 52 = 0 \quad 1$$

21. E_1 : student knows the answer

E_2 : student guesses the answer

A: answers correctly.

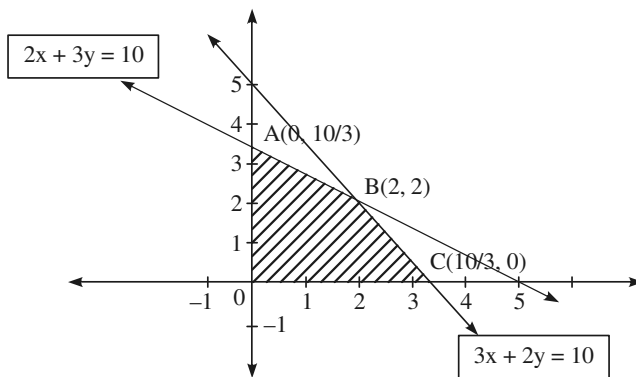
$$P(E_1) = \frac{3}{5}, \quad P(E_2) = \frac{2}{5} \quad 1$$

$$P\left(\frac{A}{E_1}\right) = 1, \quad P\left(\frac{A}{E_2}\right) = \frac{1}{3} \quad 1+1$$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)} \quad 1$$

$$= \frac{\frac{3}{5} \cdot 1}{\frac{3}{5} \cdot 1 + \frac{2}{5} \cdot \frac{1}{3}} = \frac{9}{11} \quad 1+1$$

22.



L.P.P. is Maximise $P = 24x + 18y$ $\frac{1}{2}$

s.t. $2x + 3y \leq 10$ $\left. \begin{array}{l} 3x + 2y \leq 10 \\ x, y \geq 0 \end{array} \right\} 2$

$3x + 2y \leq 10$ $\left. \begin{array}{l} x, y \geq 0 \end{array} \right\} 2$

$x, y \geq 0$ $\left. \begin{array}{l} \text{Correct figure} \\ P(A) = \text{Rs } 60 \\ P(B) = \text{Rs } 84 \\ P(C) = \text{Rs } 80 \end{array} \right\} 2$

Correct figure $\left. \begin{array}{l} P(A) = \text{Rs } 60 \\ P(B) = \text{Rs } 84 \\ P(C) = \text{Rs } 80 \end{array} \right\} 2$

$P(A) = \text{Rs } 60$ $\left. \begin{array}{l} P(B) = \text{Rs } 84 \\ P(C) = \text{Rs } 80 \end{array} \right\} \frac{1}{2}$

$P(B) = \text{Rs } 84$ $\left. \begin{array}{l} P(C) = \text{Rs } 80 \end{array} \right\} \frac{1}{2}$

$P(C) = \text{Rs } 80$ $\left. \begin{array}{l} \therefore \text{Max.} = 84 \text{ at } (2, 2) \end{array} \right\} 1$

$\therefore \text{Max.} = 84 \text{ at } (2, 2)$ 1

23. Given: $s = 4\pi r^2 + 2\left[\frac{x^2}{3} + 2x^2 + \frac{2x^2}{3}\right]$

$$= 4\pi r^2 + 6x^2 \quad 1$$

$$V = \frac{4}{3}\pi r^3 + \frac{2x^3}{3}$$

$$V = \frac{4}{3}\pi r^3 + \frac{2}{3}\left(\frac{S - 4\pi r^2}{6}\right)^{3/2} \quad 1$$

$$\frac{dv}{dr} = 4\pi r^2 + \left(\frac{S - 4\pi r^2}{6}\right)^{1/2} \left(\frac{-8\pi r}{6}\right) \quad 1$$

$$\frac{dv}{dr} = 0 \Rightarrow r = \sqrt{\frac{S}{54 + 4\pi}} \quad 1$$

showing $\frac{d^2v}{dr^2} > 0$ 1

\therefore For $r = \sqrt{\frac{S}{54 + 4\pi}}$ volume is minimum

i.e., $(54 + 4\pi)r^2 = 4\pi r^2 + 6x^2$

$6x^2 = 54r^2 \Rightarrow x^2 = 9r^2 \Rightarrow x = 3r$ 1

24. Here,

$$R = \left\{ \begin{array}{l} (1,1), (2,2), (3,3), (4,4), (5,5) \\ (1,3), (1,5), (2,4), (3,5) \\ (3,1), (5,1), (4,2), (5,3) \end{array} \right\} \quad 2$$

Clearly

(i) $\forall a \in A, (a, a) \in R \quad \therefore R$ is reflexive 1

(ii) $\forall (a, b) \in A, (b, a) \in R \quad \therefore R$ is symmetric 1

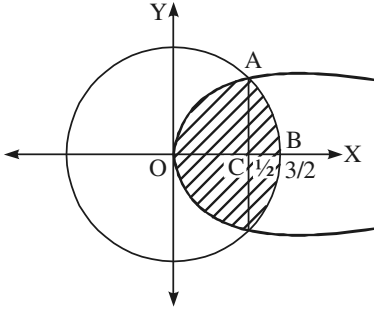
(iii) $\forall (a, b), (b, c) \in R, (a, c) \in R \quad \therefore R$ is transitive 1

$\therefore R$ is an equivalence relation.

$[1] = \{1, 3, 5\}, \quad [2] = \{2, 4\}$ 1

25.

$$\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$$



Correct figure

1

Getting $x = \frac{1}{2}$ as point of intersection

1/2

$$A = 2 \left[2 \int_0^{1/2} \sqrt{x} dx + \int_{1/2}^{3/2} \sqrt{\frac{9}{4} - x^2} dx \right]$$

1

$$= 2 \left[\left(\frac{4}{3} x^{3/2} \right)_0^{1/2} + \left(\frac{x}{2} \sqrt{\frac{9}{4} - x^2} + \frac{9}{8} \sin^{-1} \frac{2x}{3} \right)_{1/2}^{3/2} \right]$$

1 1/2

$$= 2 \left[\frac{2}{3\sqrt{2}} + \frac{9\pi}{16} - \frac{\sqrt{2}}{4} - \frac{9}{8} \sin^{-1} \frac{1}{3} \right]$$

1

$$= \frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{3} \text{ sq. unit}$$

1

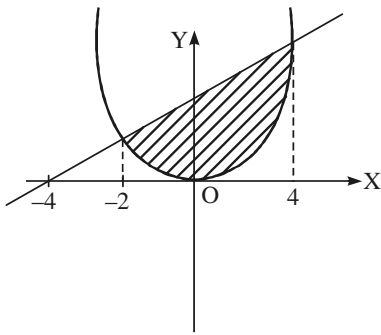
OR

Correct figure

1

Getting $x = 4, -2$ as points of intersection

1/2



$$A = \int_{-2}^4 \frac{1}{2} (3x + 12) dx - \int_{-2}^4 \frac{3}{4} x^2 dx$$

1

$$= \frac{1}{2} \left(\frac{3x^2}{2} + 12x \right)_{-2}^4 - \frac{1}{4} (x^3)_{-2}^4$$

1 1/2

$$= \frac{1}{2} (24 + 48 - 6 + 24) - \frac{1}{4} (64 + 8)$$

1 1/2

$$= 45 - 18 = 27 \text{ sq. units}$$

1/2

26. $\left(x \sin^2 \left(\frac{y}{x} \right) - y \right) dx + x dy = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{y - x \sin^2(y/x)}{x} = \frac{y}{x} - \sin^2\left(\frac{y}{x}\right) \quad 1$$

$$v + x \frac{dv}{dx} = v - \sin^2 v \quad \text{where } \frac{y}{x} = v. \quad 1$$

$$\Rightarrow \int -\frac{dv}{\sin^2 v} = \int \frac{dx}{x} \quad \text{or} \quad \int -\operatorname{cosec}^2 v \, dv = \int \frac{dx}{x} \quad 1\frac{1}{2}$$

$$\cot v = \log x + C \quad \text{i.e.,} \quad \cot \frac{y}{x} = \log x + C \quad 1\frac{1}{2}$$

$$y = \frac{\pi}{4}, x = 1, \Rightarrow C = 1 \quad \frac{1}{2}$$

$$\Rightarrow \cot \frac{y}{x} = \log x + 1 \quad \frac{1}{2}$$

OR

$$\frac{dy}{dx} - 3 \cot x \cdot y = \sin 2x$$

$$\text{IF} = \int -3 \cot x \, dx = -3 \log \sin x = \operatorname{cosec}^3 x \quad 1$$

∴ Solution is

$$y \cdot \operatorname{cosec}^3 x = \int \sin 2x \operatorname{cosec}^3 x \, dx \quad 1\frac{1}{2}$$

$$= \int 2 \operatorname{cosec} x \cot x \, dx \quad \frac{1}{2}$$

$$y \cdot \operatorname{cosec}^3 x = -2 \operatorname{cosec} x + C \quad 1\frac{1}{2}$$

$$\text{or } y = -2 \sin^2 x + C \sin^3 x$$

$$x = \frac{\pi}{2}, y = 2 \Rightarrow C = 4 \quad 1$$

$$\Rightarrow y = -2 \sin^2 x + 4 \sin^3 x \quad \frac{1}{2}$$